TR-EE 65-11

FINAL REPORT

Volume II

covering the period
1 July 1964 to 30 July 1965

prepared for

George C. Marshall Space Flight Center, Huntsville, Alabama

National Aeronautics and Space Administration

on

Contract NAS 8-11416

PRF 4005-52-285

titled

ADVANCED CONTROL TECHNIQUES APPLIED TO LARGE FLEXIBLE LAUNCH VEHICLES

with

Control and Information Systems Laboratory

School of Electrical Engineering

Purdue University

Lafayette, Indiana

J. E. Gibson, Principal Investigator

30 July, 1965

Advanced Control Techniques Applied to

Large Flexible Launch Vehicles

by

J. E. Gibson

J. C. Hill

V. Haas

A. S. Morse

S. Murtuza

L. E. Jones III

A. Steinberg

July 1965

LIST OF FIGURES

CHAPTER VI

ADAPTIVE CONTROL APPLIED TO A FLEXIBLE VEHICLE

6.1 Introduction

In man's efforts to conquer space he is using larger and larger launch vehicles. As vehicles are increased in length, without equivalent increases in structural rigidity, the values of their bending mode frequencies are decreased. As these frequencies decrease into the range of the control frequencies the control problem becomes more difficult.

This chapter is an introduction to the use of adaptive control concepts to solve this problem. It surveys the state of the art, including adaptive techniques applicable to the control of Model Vehicle No. 2. A lengthy annotated bibliography is given in Appendix A.

In the ensuing chapter the mathematical model for a large launch vehicle such as MV2 is discussed. Assumptions employed in the derivations are discussed and some weaknesses in the model are pointed out.

Shortcomings of rigid body control for MV2 are considered and the conclusion reached that the lower flexible modes (first and second) must be controlled actively.

Based on the uncertainty of the model and on the wide diversity of environmental conditions it is further concluded that some form of adaptive or self-adjusting control system would be desirable.

The flexible vehicle control problem is related closely to signal discrimination or state estimation. That is, all sensors on the vehicle measure a combination of the rigid body and bending body signals. A large portion of the control problem consists of discriminating between the different components of the measured signals.

The tracking notch filter and the digital adaptive filter are considered as avenues of approach to solution of the discrimination problem. It is concluded that the former is inapplicable for the first and second bending modes of the vehicle. The latter is extremely complex and appears undesirable, especially in comparison with the final technique considered.

Model reference adaptive control appears to be the most versatile of all schemes considered. Choice of the reference model and the form of the controller allows one to include all the a priori knowledge available. Then the self-adjustment of the parameters corrects for errors in the system coefficient values.

Response to disturbances is not a solved problem and this is one possibility for further work. Conclusions and recommendations are made in the final section of the chapter.

6.2 Discussion of the Mathematical Model

Before considering a controller for the vehicle it is necessary to investigate the existing mathematical model of the vehicle. It is especially important to know the weaknesses in the model so design adjustments can be made to reduce their effects. Model Vehicle No. 2 (MV2) has been furnished [1] for use in control studies. Appendix B includes some portions of the derivation of the equations given in [1].

Essentially the vehicle equations are derived under the following assumptions:

1. The mass, thrust, velocity, etc., of the vehicle vary slowly with respect to transients in the control loop, and therefore are assumed to be quasi-stationary.
2. The vehicle is flying a gravity turn trajectory and the equations are the linearized perturbation equations about a nominal trajectory.
3. The attitude control will maintain the vehicle sufficiently close to the nominal trajectory that small angle assumptions are valid.
4. For the purpose of describing flexibility, the vehicle is assumed to be a free-free beam. Bending is then superposed on the rigid body.
5. Slosh phenomena can be described by spring-mass systems properly located along the vehicle.

For the ramainder of this chapter slosh, actuator dynamics, and engine inertial effects ("tail-wags-dog") will be neglected. This is not done without realizing that these effects do play a large role in the overall response of the vehicle. For the present, though, it is assumed that the vehicle has ideal baffles, actuator dynamics, and a zero-inertia gimbal system. Certainly for MV2 these effects play a lesser role than does bending.

In addition to the errors in the equations contributed by assumption 1, the aerodynamic properties of MV2 are altered considerably during flight due to extreme changes in velocity and altitude. So the aerodynamic moment coefficient C_1 changes widely (see Appendix B for curve) and the value at any time is questionable. The variations of mass, center of gravity, thrust, and moment of inertia make it difficult to evaluate the control moment coefficient C_2, so it too is questionable. Reference [1] suggests that a $\pm$ 20% tolerance be considered for C_1 and C_2.

Assumption 4, though it may appear questionable at first glance, is not entirely without justification. Through experience, reasonably good analytical techniques have been developed for predicting the bending frequencies and normal mode shapes via a beam equation analysis. However, errors are present. Superposition of the flexible body on the rigid body is valid only for slight bending; i.e., the lateral deflection or bending must be small enough that the motion in the direction parallel to the vehicle axis is negligible.

Appendix B includes a derivation of the bending moment of MV2. The expression is valid only for small angles.

To summarize, the vehicle equations contain coefficients whose values are indefinite. In designing a controller the effects of errors in assumed values should be minimized. A means of accomplishing this is to use an adaptive controller.

6.3 Conventional Control

This section considers some of the conventional techniques for attitude control of a launch vehicle. Their relation to the control of MV2 is discussed.

6.3.1 Rigid Body Control

An early approach to the design of an attitude control system for a launch vehicle was to assume that the vehicle was rigid. For this assumption to be valid there must be a sufficient separation between the rigid body control frequencies and the first bending mode frequency. Geissler [2] points out that the amount of separation needed is difficult to determine exactly, but indicates that factors of 3, 5, or even 10 may be reasonable.

Rigid body control frequency bandwidth is chosen around 0.2 to 0.9 cps. The first bending frequency of MV2 falls in the range 0.343 to 0.464 cps during the flight. At the outset one should note that ignoring the bending of MV2 is not reasonable. The first (and perhaps the second) bending should be controlled actively. Nevertheless it is worthwhile to discuss a few of the rigid body control methods.

First, consider some of the problems faced by the engineer. The aerodynamically unstable vehicle is buffeted by winds of an unknown nature. The vehicle must fly on or near its nominal trajectory without breaking from excessive structural loading. A bounded control effort is available to reduce lateral drift, attitude angle, and bending moment. These will often be conflicting interests.

The two most often discussed [2,3,4] rigid body control methods are the so-called "Drift Minimum Principle," DMP, and "Load Minimum Principle," LMP. The discussion below will be with respect to the rigid body equations only, which are

$$\begin{aligned} \ddot{\phi}_R &= -C_1\alpha - C_2\beta_R \\ \ddot{z} &= C_3\alpha + C_4\beta_R + C_5\phi_R \\ \alpha &= \phi_R - C_7\dot{z} + \alpha_w \end{aligned} \tag{6.1a}$$

and a linear control law, assuming ideal actuator and sensors, is

$$\beta_R = a_o\phi_R + a_1\dot{\phi}_R + b_o\alpha. \tag{6.1b}$$

The bending moment for the rigid body can be expressed as

$$BM_R(x) = K_1(x)\alpha + K_2(x)\beta_R \tag{6.1c}$$

All terms are defined in Appendix B.

Assuming all C_i constant, the transfer function from V_w to $\dot{z}$ is

$$\frac{\dot{z}}{V_w}(s) = \frac{-s[s^2 + a_1C_2s + C_1 + (a_o + b_o)C_2] + P(s)}{P(s)} \tag{6.2}$$

where the characteristic equation is

$$\begin{aligned} P(s) = s^3 &+ [a_1C_2 + C_7(C_3+b_oC_4)]s^2 \\ &+ [C_1 + (a_o + b_o)C_2 + a_1C_7(C_2C_3 - C_1C_4)]s \\ &+ C_7[a_o(C_2C_3 - C_1C_4) - C_5(C_1 + b_oC_2)] \end{aligned} \tag{6.3}$$

Assuming the feedback gains have been chosen to yield a stable controller, the final value theorem can be used to examine the steady state drift rate for a constant wind velocity. It is found to be $\dot{z}_{ss} = V_w$ (the constant wind velocity). Setting the coefficient of the zero power of s in the characteristic equation to zero yields the condition

$$b_o = \frac{C_2C_3 - C_1C_4}{C_2C_5}\ a_o - \frac{C_1}{C_2} \tag{6.4}$$

A stable controller satisfying this relationship yields

$$\dot{z}_{ss} = \frac{a_1C_7(C_2C_3 - C_1C_4)}{(a_o + b_o)C_2 + C_1 + a_1C_7(C_2C_3 - C_1C_4)}\ V_w \tag{6.5}$$

Examining Eq. (6.5) it can be seen the $\dot{z}_{ss}$ can be less than V_w by choosing

$$(a_o + b_o)C_2 + C_1 > 0 \tag{6.6}$$

Therefore it is defined that the DMP is that control mode in which Eq. (6.4) is satisfied.

If b_o is larger than the right hand side of Eq. (6.4) the control is unstable, so it is advisable to use slightly less than the equality for b_o to protect against the possibilities of inexact parameter values.

The effect of using the DMP is that the vehicle turns itself so that the components of force due to aerodynamic and control inputs cancel, thus reducing the drift.

As the vehicle passes through mach 1 and the regions of jet winds and maximum aerodynamic pressure (max q) a different approach is needed for the controller. By examining the bending moment given by Eq. (6.1c) one can make the following observations. Provided the controller is maintaining the vehicle approximately on the nominal trajectory the value of ϕ_R will be reasonably small and well damped. Even if b_o is large enough to make the system "slowly unstable", ϕ_R will build up slowly at first. Furthermore c_7 has by this time become small so we can say that

$$\alpha \approx \alpha_w \tag{6.7}$$

Especially in the case of extremely large wind disturbances one can see that the bending moment is highly dependent on α. This gives rise to the LMP, or angle-of-attack control, in which $a_o = 0$. This mode is an unstable mode except for the smallest of b_o values, which would be inadequate because small b_o implies slow response time. The LMP cannot be applied over extended periods of time, for ϕ, $\dot{\phi}$, and $\dot{z}$ would increase to a point at which the above reasoning becomes invalid.

Therefore the DMP tends to reduce the lateral drift rate and the LMP tends to reduce angle-of-attack and, consequently, the bending moment. These control modes are not arrived at by optimal control techniques so the term "minimum" is a misnomer. The reasoning behind the DMP relies heavily on steady state response to a constant wind. Unless the control bandwidth is high with respect to the frequency of the changing wind velocity, much of this reasoning is invalid. The LMP turns the vehicle into the wind in order to decrease aerodynamic loads, leaving ϕ free to do as it will.

Several disadvantages are incurred in these control modes. The DMP requires that Eq. (6.4) be satisfied at all times. This is a preprogrammed controller which will cause difficulties because of the uncertainty of the values of the C_i. The LMP leads to instability and is applicable only for short periods of time. Neither of these have accounted for the flexure of the vehicle.

Provided the bending of the vehicle can be maintained with reasonably small limits the following approach might be considered. Use the DMP during the early part of boost, change to LMP during the region of jet winds and max q, then change back to the DMP afterwards. Excessive departures from the nominal trajectory during use of the LMP could be corrected by maneuvers commanded for the guidance loop. This gives encouragement to the idea of using different performance indices, or a time variable PI.

Further discussion of the DMP and LMP, also referred to as Minimum Drift Control (MDC) and Minimum Load Control (MLC) is given in Appendix E.

6.3.2 Flexible Body Control

Once the rigid body controller has been decided upon it is necessary to examine the effects of bending. The early attempts to reduce bending fall into the categories of "gain stabilization" and "phase stabilization". "Gain stabilization" in essence consists of low pass filtering all sensed signals to remove the bending signal. For this to operate, it is necessary to have separation of control and bending frequencies. Another means of "gain stabilization" is by placing the sensors in a manner so as to measure only the rigid signal. For instance, by placing the rate gyro at an antinode it senses only the rigid body portion, provided this is the only bending mode excited. Drawbacks to this are the movement of the antinode during flight and the presence of additional modes.

"Phase stabilization" consists of feeding back bending signals with a phase relationship leading to increased damping; specifically, one may mount a rate gyro aft of the antinode to assure proper phase for the first bending mode.

For MV2 these general schemes are applicable in various forms, however to achieve either by proper placement of a sensor is doubtful due to the uncertainty of parameter values. The next section discusses several approaches more applicable to MV2.

6.4 Adaptive Control

The concept of adaptive control, as used here, is explained below. Reasons for needing an adaptive controller for MV2 are given. Several possible techniques from the current literature are discussed in relation to the control of MV2.

6.4.1 Need and Definition

The discussion in section 6.2 of the mathematical model or vehicle equations for MV2 came to the conclusion that the form of the equations, while not exact because of linearization, is a reasonably good assumption. The model becomes very complex and of high order when several bending modes are included. The slosh, actuator dynamics, and motor inertia are still not neglected here; they too add to the complexity. The parameter values are time varying and any numbers arrived at for them are highly questionable.

During its flight MV2 is buffeted by winds of unpredictable magnitude and direction. These winds can have large effects on the vehicle: excessive drift off of the nominal trajectory, excitation of bending oscillations on the vehicle, and large bending loads which could destroy the vehicle or cause an abortion of the mission.

Therefore the problem is to design a controller which will fly the vehicle along its nominal trajectory without exceeding allowable bending moment and control angle requirements and furnished with the following information (or misinformation).

1. Vehicle dynamics of highly questionable accuracy.
2. Unknown disturbance inputs (winds).
3. Widely changing environment and plant, which will require a versatile controller.

A conventional or fixed controller is designed to perform well within a range of conditions of environment, inputs, and disturbances. There is usually a trade off between how well and the size of the range. A conventional controller usually behaves poorly under conditions other than those designed for. The conditions associated with MV2 vary over a very large range.

Consequently the possibility of a prescheduled or programmed controller should be considered. In such a controller, the pertinent information about the conditions the system faces are measured. Then the correct controller is chosen from a previously prepared table. The DMP falls under this general category. The designer must have a good set of equations to work with, and needs to make extensive tests on the system to insure the proper gain combinations are chosen. For MV2 the equation parameters are questionable and flight tests are infeasible due to the expense.

The obvious conclusion is that for MV2, because of its high degree of flexibility and for other reasons discussed above, an adaptive control system is desirable.

Gibson [5] defines an adaptive control system as one which compares its performance to that prescribed by an index of performance and adjusts itself toward the optimum according to this criterion. In doing so it performs the three functions of identification, decision, and modification.

In this discussion the classification is broadened somewhat. An adaptive or self-adjusting control system is one which includes any means of automatic adjustment which improves the control performance. The differences and similarities between these definitions can furnish material for endless debate.

In general, any adaptive control system depends heavily on the performance index. Often the control objective can only be stated in vague words. It is extremely difficult to translate this into a realistic PI. For MV2 it is required to fly along the nominal trajectory without breaking up and using only the available control. This is obviously a vague statement and it is difficult to formulate it as a precise integral or final value performance index. <u>This is the weakest link in any optimization problem</u>. Since adaptive control is effectively on-line optimization this remains a weak link.

Reducing or eliminating bending is a necessary requirement for the

MV2 controller. The bending moment must be maintained within safe limits and the control effort is bounded. The drift, attitude angle, and their rates are especially important at the end of the flight, though if one considers the entire set of vehicle equations rather than the perturbations about the nominal trajectory it is not entirely obvious that these quantities (especially z and ϕ) should be kept small at all times.

At this point no specific index of performance will be offered. Instead the discussion will be directed toward possible adaptive techniques applicable to the problem. Andeen [6,7] discusses five categories of adaptive techniques for stabilization of large, flexible vehicles. They are 1) rate-gyro blending, 2) tracking notch filters, 3) multiple sensors, 4) model reference/implicit systems, and 5) rigid body separation. Rate-gyro blending, actually a subcategory of multiple sensors, consists of mounting two rate-gyros so that one is fore, the other aft of the first mode antinode. The signals are added and the relative gain of the two is adjusted to either eliminate the bending (for gain stabilization) or feed it back with the proper phase for increasing the damping of the bending mode (phase stabilization).

Below other techniques are discussed with relation to their application to the control of MV2.

6.4.2 Tracking Notch Filter

All sensors mounted on a flexible vehicle, such as MV2, measure a combination of the rigid body and flexible body information. This combination varies along the vehicle as the normalized mode shapes and at any station it varies with time of flight. It is necessary to obtain separate signals for the rigid body and flexible body from the measured signal if one insists on superposition of bending corrections on a rigid body analysis. One must actually separate the flexible signal into components for each bending mode. Once the engineer has at his disposal the separate signals (or equivalently, knowledge of the combination) he may combine them in such a manner as to effectively control the vehicle. In essence the engineer is faced with a difficult problem of signal discrimination or state estimation.

The problem may be stated in terms of MV2 for one bending mode as follows. The measured signal ϕ is a summation of the two states ϕ_R and η_1.

$$\phi\ (t,x) = \phi_R(t) - Y_1'(t,x)\ \eta_1(t) \tag{6.8}$$

where $Y_1'(t,x)$ is the slope of the first normalized mode shape at time t and station x. Predominantly ϕ_R is a damped second order sinusoid

$$\phi_R(t) = e^{-at}(A \sin \omega_R t + B \cos \omega_R t) \tag{6.9}$$

and the bending signal is an undamped second order sinusoid

$$\eta_1(t) = C \sin \omega_{B_1} t + D \cos \omega_{B_1} t \tag{6.10}$$

The problem is to obtain ϕ_R and η_1 from ϕ. It is even more complicated since there are also higher bending mode signals and measurement noise in ϕ.

One approach for separating these signals is frequency discrimination. For a case where ω_{B_1}/ω_R is sufficiently high they may be separated by low pass filtering to obtain ϕ_R. No precise number can be given for ω_{B_1}/ω_R but [2] does mention the possibilities of 3, 5, or even 10 being reasonable. As ω_{B_1} and ω_R approach each other low pass filtering becomes inadequate. The control problem also becomes more difficult. With wide separation the bending can be neglected provided only ϕ_R and other rigid body signals are fed back.

A sharply tuned (narrow rejection band) notch filter can be used to reject the ω_{B_1} signal. For ω_{B_1} and ω_R close together, the rejection band must be made narrow enough not to interfere with the ω_R signal. For MV2 these frequencies are very close and possibly ω_{B_1} could be less than ω_R at some times. A scheduled tuning of a notch filter is still feasible provided that a sharp notch is available and accurate a priori knowledge of ω_{B_1} is available throughout the flight.

Earlier discussions emphasize that although analytical techniques are quite advanced, an exact tabulation of ω_{B_1} is not possible. The rejection band or notch must be very narrow for the first bending mode of MV2 to reduce interference with the rigid body signal. So a slight error in the tabulation could result in passing the bending signal virtually undisturbed through the filter for the entire flight. No separation would be obtained in this case.

A tracking notch filter can be used to eliminate the need for tabulating ω_{B_1}. The block diagram of such a unit is depicted in Figure 6.1. A transfer function representation is used though it should be recognized that this is really invalid when ω_E, the current estimate of ω_{B_1}, varies. The input x is passed through a band pass filter tuned to the current value of ω_E. This allows the frequency tracking unit to operate on a signal containing primarily the frequency of interest.

Several frequency tracking units are discussed in the literature [6,8] and one example is depicted in Figure 6.2. The operation of this unit is as follows. The signal y is assumed to be an undamped sinsoid

$$y = A \sin \omega_{B_1} t \tag{6.11}$$

with second derivative

$$\ddot{y} = -\omega_{B_1}^2 A \sin \omega_{B_1} t = -\omega_{B_1}^2 y \tag{6.12}$$

The following relationship is obviously true

$$\omega_{B_1}^2 |y| = \omega_{B_1}^2 A = |\ddot{y}| \tag{6.13}$$

It is desired to adjust $b=\omega_E^2$, the estimate of $\omega_{B_1}^2$, to minimize e^2, where

$$e = |\ddot{y}| - b|y| = A(\omega_{B_1}^2 - b) \tag{6.14}$$

The adjustment equation which changes b in the direction of steep descent is

$$\frac{db}{dt} = -k \frac{\partial e^2}{\partial b} \tag{6.15}$$

Substituting Eq. (6.14) into Eq. (6.15) yields

$$\frac{db}{dt} = 2kAe = Ke \tag{6.16}$$

and due to the arbitrary nature of k it is reasonable to lump 2kA into a new constant K, to be chosen by the designer. The lags are introduced in both paths to avoid the pure differentiation.

It is necessary that ω_{B_1} be separated from the rigid body frequencies for such a self-adjusting system to operate. Otherwise the frequency tracking unit might track the wrong signal, leading to disastrous results. It is concluded that the use of a tracking notch filter to reject the first bending signal of MV2 is inadvisable, due to insufficient difference between ω_{B_1} and ω_R.

6.4.3 Digital Adaptive Filter

At this point the prospects of frequency discrimination between rigid and flexible body signals appear unpromising. Discrimination based on frequency and damping has been suggested by Zaborszky et al [9]. The technique is described briefly below.

In general the rigid body response is more highly damped than that of the flexible body (Eqs. (6.9) and (6.10) indicate typical responses). Thus it is assumed that ϕ_R can be estimated at the n^{th} sample time by the model

$$\hat{\phi}_R(t_n) = e^{-at_n} (A_N \cos \omega_R t_n + B_N \sin \omega_R t_n) \tag{6.17}$$

For the present only a two parameter fit is discussed. A_N and B_N are the values estimated at the N^{th} sample time. The deviation from the measured signal $\phi(t)$ at the n^{th} sample time is

$$d\ (t_n) = \phi(t_n) - \hat{\phi}_R\ (t_n) \tag{6.18}$$

When $\hat{\phi}_R(t) = \phi_R(t)$, the deviation $d(t)$ is the bending signal. This technique is used as part of the attitude control loop, and the objective of the control is to feedback a signal such that the bending is reduced. The signal fed back to the controller is $\hat{\phi}_R$. During the time interval $t(NT) = t_N \leq t < t_{N+1}$, $\hat{\phi}_R$ is computed using t_N, A_N, and B_N. T is the sampling period. Using the M most recent samples of ϕ, A_N and B_N are chosen to minimize

$$I\ (A_N, B_N) = \sum_{i=1}^{M} w^2(t_i)\ d^2(t_{i+N-M}) \tag{6.19}$$

where $w(t_i)$ is a weighting function.

The necessary conditions which must be satisfied are

$$\frac{\partial I}{\partial A_N} = 0 \qquad \frac{\partial I}{\partial B_N} = 0 \tag{6.20}$$

Define the M-vecotrs U_A, U_{AW}, U_{BW}, and Φ whose i^{th} components are

$$\left.\begin{aligned}
U_{A_i} &= -W(t_i)\, e^{-at_{i+N-M}} \cos \omega_R t_{i+N-M} \\
U_{B_i} &= -W(t_i)\, e^{-at_{i+N-M}} \sin \omega_R t_{i+N-M} \\
U_{AW_i} &= W(t_i)\, U_{A_i} \\
U_{BW_i} &= W(t_i)\, U_{B_i} \\
\Phi_i &= \Phi(t_{i+N-M})
\end{aligned}\right\} \qquad (6.21)$$

Define the scalar products

$$\left.\begin{aligned}
b_{AA} &= U_A^T\, U_A \\
b_{AB} &= U_A^T\, U_B = U_B^T\, U_A \\
b_{BB} &= U_B^T\, U_B
\end{aligned}\right\} \qquad (6.22)$$

and finally define the M-vectors

$$\left.\begin{aligned}
V_A &= \frac{U_{BW} b_{AB} - U_{AW} b_{BB}}{b_{BB} b_{AA} - b_{AB}^2} \\
V_B &= \frac{U_{AW} b_{AB} - U_{BW} b_{AA}}{b_{BB} b_{AA} - b_{AB}^2}
\end{aligned}\right\} \qquad (6.23)$$

Solving Eq. (6.20) and applying all the terms defined in Eqs. (6.21), (6.22), and (6.23), it is found that the value for A_N and B_N should be

$$\begin{aligned}
A_N &= V_A^T\, \Phi \\
B_N &= V_B^T\, \Phi
\end{aligned} \qquad (6.24)$$

The system operates on the measured signal only during transient periods. Means are included for sensing a transient. Then a fade-in period occurs during which M increases from 1 to the desired value. During this time the computer is storing sample points. From the time of the initiation of the new transient, the controller receives the estimated signal $\hat{\phi}_R$ for use as feedback. Since the current value of $\hat{\phi}_R(t_N)$ is calculated only after the computation of A_N and B_N we should consider Eqs. (6.24). Note that the V_A and V_B vectors are independent of ϕ and can be precomputed and stored. Then it is necessary to insert the newest measurement $\phi(t_N)$, shifting out the oldest, and to compute A_N and B_N. The computation time enters as a transportation lag. This can be partially eliminated by employing an extrapolation on the A_N and B_N values.

It is also possible to use a four parameter fit, in which a and ω_R are estimated. The computation per stage is considerably higher.

Several observations can be made concerning this technique. It is a complex technique requiring: 1) sensing of the beginning of a transient, 2) estimation during fade-in, 3) extrapolation to reduce the effect of lag for computation, and 4) a digital computer with A to D and D to A converters. The concept of importance here is the discrimination between rigid body and bending body signals based on damping and frequency alone. Assuming $\hat{\phi}_R$ as given by Eq. (6.17) is equivalent to assuming a model for the rigid body signal. Then this model is adjusted to more nearly represent the true case.

For MV2 the flexible modes are lightly damped. With a controller placed on the vehicle the damping of the bending modes is increased. Therefore, here as in other techniques, the rigid body and flexible body signals have similar characteristics. Application of this technique to the control of MV2 would be a difficult problem due to the complexity of the technique.

Several recent contributions have been made in the area of state estimation for nonlinear systems. Sridhar et al [10] and Detchmendy [11] are two examples. The prospects of a sequential estimation scheme are considered and found promising. At this time a considerable amount of digital solution is required in the use of these techniques.

6.4.4 Model Reference Adaptive Control, MRAC

The tracking notch filter is a means of separation of rigid and bending signals by frequency discrimination. The digital adaptive filter assumes a form or model for the rigid body feedback and adjusts this model to minimizing the bending. In this respect the digital adaptive filter is similar to a model reference adaptive controller, MRAC. A review of the design capabilities of MRAC by Whitaker [12] and papers by Kezer, Hofmann, and Engel [13], Clark [14] and others [6, 15] discuss several applications of MRAC.

For the MRAC the goals or performance criteria of the system are incorporated into a reference model. This real-time model is driven in parallel with the controlled system. A feedback controller is specified except for the parameters to be self-adjusted. The response error is formed from the difference between the desired output from the model and the system output. Certain parameters of the system are adjusted in a closed-loop fashion to minimize the short term average of the squared error. A block diagram of a general MRAC system is given in Figure 6.3.

The formulation of an index of performance following the line of the discussion in Section 6.4.1 is not required. Instead a model must be chosen which characterizes the desired response to the command input. Reference [13] discusses the relationship between the reference model and time and frequency

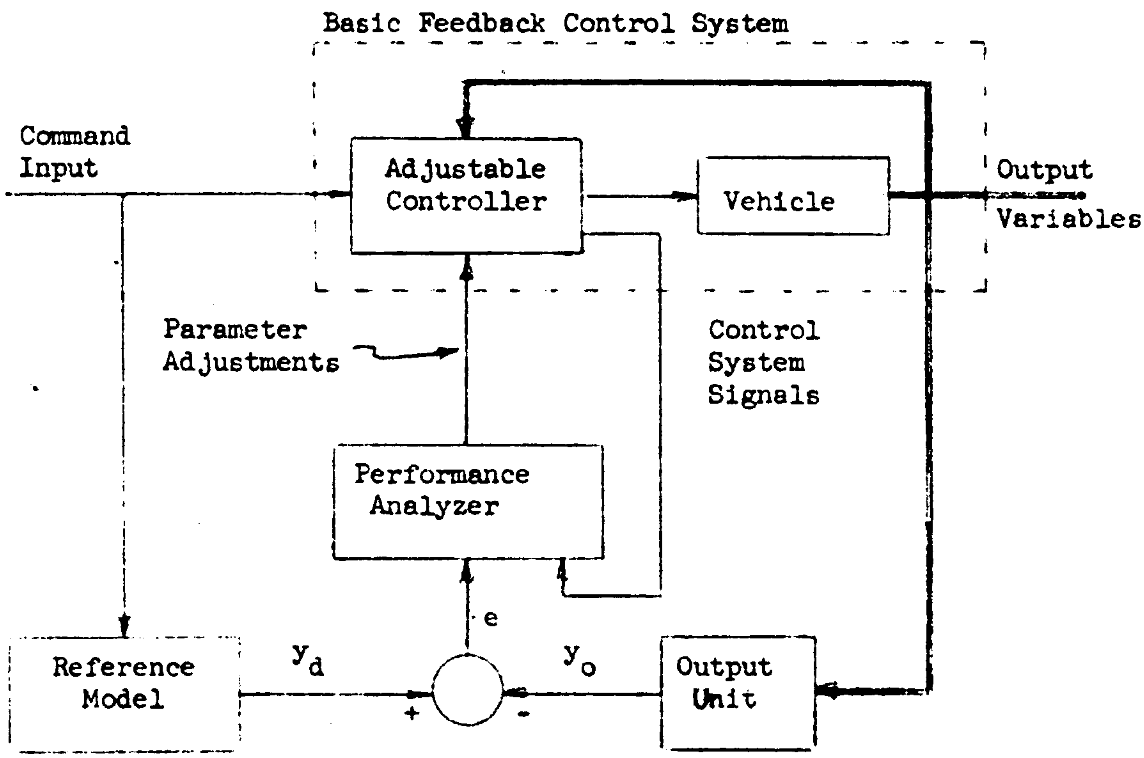


Figure 6.3

A Model Reference Adaptive Control System domain performance criteria.

The problem of identification of the system is one which need not be solved exactly for MRAC. The engineer needs and can use all the a priori knowledge at hand, but it is only necessary to be sure that the form of the self-adjusting controller is versatile enough to conform the system output to that of the model. Obviously the more information known a priori the better the design will be.

Once the reference model and the form of the controller have been chosen the output y_o from the system must be obtained. For example, in Reference [13] the reference model was chosen to be an open circuit between command input and first bending mode response, thus

$$y_d = \eta_{1d} \equiv 0 \tag{6.25}$$

No direct measure of $\dot{\eta}_1$ is available so the output unit consists of two rate-gyros, one fore (x_F), the other aft (x_A) of the first antinode. Then in terms of MV2 with first bending mode,

$$\begin{aligned} \dot{\phi}(x_A) &= \dot{\phi}_R - Y_1'(x_A)\,\dot{\eta}_1 \\ \dot{\phi}(x_F) &= \dot{\phi}_R - Y_1'(x_F)\,\dot{\eta}_1 \end{aligned} \tag{6.26}$$

And $Y_1'(x_A) > 0$, $Y_1'(x_F) < 0$ which allows us to take the difference to obtain

$$K\dot{\eta}_1 = [Y_1'(x_A) - Y_1'(x_F)]\,\dot{\eta}_1 = \dot{\phi}(x_F) - \dot{\phi}(x_A) \tag{6.27}$$

and integrating we get

$$y_o = K\,\eta_1 \tag{6.28}$$

The adjustable parameter is the relative gain on the two rate measurements which are summed and used in the control loop. This reduces to a form of rate-gyro blending wherein the relative gains of two rate-gyro measurements are adjusted to minimize the short term average of the squared response error. In this case the error is the negative bending and the parameter P is adjusted by

$$\dot{P} = -k\, e \frac{\partial e}{\partial P} \tag{6.29}$$

Because P has no effect on y_d (even it it isn't assumed zero as in this case)

$$\frac{\partial e}{\partial P} = - \frac{\partial y_o}{\partial P} \tag{6.30}$$

and this signal can be obtained by passing the signal at the input of the gain P through a filter containing the control system transfer function (assuming at least a quasistationary system). The system is unknown because no identification is performed. However, a model has been chosen which, as the control parameter approaches its optimum setting, will be closely approximated by the system. Therefore this filter can be approximated by the model.

Several disadvantages arise in the use of MRAC techniques. The rate of adjustment of the parameter depends on the error. As the error becomes small the parameter changes slowly. Then if a large disturbance or command input enters the system $\dot{P}$ can become very large. The system behavior was examined assuming $\dot{P}$ small, so that transfer functions could be used. For $\dot{P}$ too small, the system is sluggish. Reference [14] includes a study of this problem and suggests a peak holding technique for its solution. That is, when a large value of $\dot{P}$ occurs it is stored on a capacitor and is discharged slower than the transient of the system signals. The time constant of the holding circuit is another parameter at the designer's disposal.

Behavior in the face of disturbance inputs has not been sufficiently studied. When disturbances enter the system the parameters adjust so as to nullify their presence, because the model is unaffected and y_o must follow y_d. What is happening to other variables in the system is questionable. This is a drawback to characterizing the response of a complicated system by only one output. Then (with the parameters off their optimum setting for following command inputs) when the disturbances shift or cease the system must readjust to again reach its optimum.

In spite of the disadvantages discussed above, model reference adaptive control is the most promising technique mentioned here. It has the advantage of being a closed loop system even if the adjustment loops fail. Also an engineer can bring to bear all his a priori knowledge in choice of the reference model and the form of the controller. The MRAC technique is extremely versatile and it is recommended that it be studied further.

6.5 Conclusions and Proposed Extensions

Naturally an exact description of the vehicle to be controlled would be helpful. However, since an engineer can never expect such a situation (MV2 is typical in this regard; C_1 and C_2 can vary by $\pm$ 20% at any time in flight), a control scheme based upon such knowledge is probably impractical.

The rigid body modes of control should only be applied to MV2 after considerable care has been taken to ascertain the effects of bending. It appears that the first bending mode should be controlled actively, thus changing the DMP. The LMP is reasonable during flight through regions of high aerodynamic loading.

Inherent in the MV2 control problem is a problem of signal discrimination. Once the rigid body and flexible body signals are separated, it is possible to design a controller which damps out the bending while satisfying the other system requirements. Separation by low pass filtering is impractical for the first bending mode because ω_{B_1} and ω_R will most likely be too close together. A preprogrammed notch might reject the first bending signal with little effect on the rigid body information. This preprogramming would require accurate knowledge of ω_{B_1} during flight. A tracking notch filter to reject the ω_{B_1} signal could run into difficulties because of the close proximity of ω_{B_1} to ω_R.

A means of partially circumventing this signal separation problem may be to use a complete adaptive control system. Not only is a frequency tracked, but the controller itself adjusts for optimal performance. The most versatile are the model reference adaptive control techniques. In this configuration the system is a closed-loop control system in which certain parameters adjust themselves automatically.

To apply model reference techniques to MV2 it is necessary to choose a reference model whose output characterizes the desired response. System behavior is the presence of disturbances is an unsolved problem. For instance, the controller tends to maintain the response error small, but other variables may not be. Some measure of the importance of this effect should be obtained.

REFERENCES

1. "Model Vehicle No. 2 For Advanced Control Studies," Working Papers furnished by M.S.F.C.

2. E. D. Geissler, "Problems in Attitude Stabilization of Large Guided Missiles," Aerospace Engineering, Oct., 1960, pp. 24-29, 68-72.

3. G. W. Johnson, F. G. Kilmer, "M.A.R.P. Report on Ballistic Missile Flight Control Theory," Sections I-V, IBM Report No. 60-508-48, Jan. 26, 1961.

4. R. F. Hoelker, "Theory of Artificial Stabilization of Missiles and Space Vehicles with Exposition of Four Control Principles," NASA-TN-D-555, June 1961.

5. J. E. Gibson, Nonlinear Automatic Control, McGraw-Hill Book Company, New York, New York, 1963, Ch. 11.

6. R. E. Andeen, "Stabilizing Flexible Vehicles," Astronautics and Aeronautics, Part 1, August 1964, pp. 38-45.

7. R. E. Andeen, "Self-Adaptive Autopilots," Space/Aeronautics, April 1965, pp. 46-52.

8. R. K. Smyth and J. C. Davis, "A Self-Adaptive Control System for a Space Booster of the Saturn Class," Preprints JACC, 1962.

9. J. Zaborszky, W. J. Luedde, and M. J. Wendl, "New Flight Control Techniques for a Highly Elastic Booster," ASD Technical Report 61-231, September, 1961.

10. R. Sridhar, G. C. Agarwal, R. M. Burns, D. M. Detchmendy, E. H. Kopf, Jr., R. Mukundan, "Investigation of Optimization of Attitude Control Systems," TR-EE65-3, Control and Informations Systems Laboratory, Purdue University, January 1965.

11. D. M. Detchmendy, "Identification, Estimation, and Control of Nonlinear Systems," Ph.D. Thesis, Purdue University, June 1965.

12. H. P. Whitaker, "Design Capabilities of Model Reference Adaptive Systems," MIT Instrumentation Laboratory Report R-374, July 1962.

13. A. Kezer, L. G. Hofmann, and A. G. Engel, Jr., "Application of Model-Reference Adaptive Control Techniques to Provide Improved Bending Response of Large Flexible Missiles," Sixth Symposium on Ballistic Missile and Aerospace Technology, Vol. II, Academic Press, 1961, pp. 113-151.

14. D. C. Clark, "The Model-Reference, Self-Adaptive Control System as Applied to the Flight Control of a Supersonic Transport," Preprints of the JACC, 1964, pp. 23-28.

15. Y. T. Li and H. p. Whitaker, "Performance Characterization for Adaptive Control Systems," Proceedings of the First International Symposium on Optimizing and Adaptive Control, 1962, pp. 109-123.

APPENDIX A

ANNOTATED BIBLIOGRAPHY ON ADAPTIVE CONTROL

During compilation of this list, primary emphasis was placed on work reported in recent years. Other bibliographies are available for past years; e.g., Stromer [118 of this list]. Rather than limit the search, articles are included which do not relate directly to the Model Vehicle No. 2 control problem.

Entries are arranged by years, most recent first, and alphabetically within each year. For brevity the following abbreviations have been used.

NEC Transactions of the National Electronics Conference

JACC Preprints of the Joint Automatic Control Conference

PGAC Transactions of the Professional Group on Automatic Control of both I.R.E. and I.E.E.E.

JBE Journal of Basic Engineering of the A.S.M.E.

IFAC Transactions of the International Federation of Automatic Control Congress.

A&RC Automation and Remote Control

[1] Gul'ko, F. B.; Kogan, B. Ya.; Lerner, A. Ya.; Mikhailov, N. N.; Novesel'tseva, Zh. A., "A Prediction Method Using High-Speed Analog Computer and its Applications," A&RC, Jan. 1965, pp. 803-813.

This article deals with a method for the optimal or nearly-optimal control of a plant by means of prediction and with the design of analog prediction units.

[2] Hamza, M. H., "Synthesis of Extremum-Seeking Control Systems," A&RC, Feb., 1965, p. 1038.

A method is outlined for the design of high speed extremal controllers. The method is based on the feeding in of test signals or periodic alternation of the input signal polarity and measurement of the discontinuities produced in the corresponding derivative output signal.

[3] Medvedev, G. A., "Synthesis of Asymptotic Optimum Dual-Mode Control Systems," A&RC, Feb., 1965, p. 1050.

A modification procedure is used to solve the fundamental equations of dual mode or bistable control theory leading to a two-stage search for solution that is amenable to natural interpretation. One stage of the solution consists of finding an optimum estimate for the unknown parameters of the controlled process, called minimum loss estimate. The application of the method is illustrated by examples.

[4] Terpugov, A. F., "Optimality Criteria for Dual Control Systems," A&RC, Feb., 1965, p. 1043.

Dual control systems with incomplete information on the number of performance steps or the object characteristics are investigated. Simplified versions of the optimality criteria are proposed which lead to asymptotically optimal systems but which are simpler analytically.

[5] Aoki, M., "On Performance Losses in Some Adaptive Control Systems," JACC, 1964, p. 29.

This paper is concerned with the problem of how long a time lapse there should be between the time data starts being collected and the time actual parameter modifications are made.

[6] Cox, H., "On the Estimation of State Variables and Parameters for Noisy Dynamic Systems," PGAC, Jan., 1964, pp. 5-12.

Using an on-line real-time digital computer, a probability function, and dynamic programming a technique is developed to estimate the states of a system.

[7] Haas, V. B., "Large Signal Adaptation for Multiple Input Plants," PGAC, January, 1964, pp. 39-46.

Concerned with obtaining optimal action by varying more than one parameter. Uses a passive controller and a controller which adapts to changing plant parameters; the adaptive controller switches relays. Applied to linear plants with constrained inputs.

[8] Hill, J. D. and McMurtry, G. J., "An Application of Digital Computers to Linear System Identification," PGAC, Oct. 1964, pp. 536-538.

Discrete interval binary noise perturbation signal used with cross correlation to identify the plant impulse response.

[9] Ho, Y. C. and Lee, R. C. K., "A Bayesian Approach to Problems in Stochastic Estimation and Control," PGAC, Oct., 1964, pp. 333-339.

Estimates the states of a system where the K^{th} estimate is based on all preceding estimates.

[10] Horowitz, I. M., "Linear-Adaptive Flight Control Design for Re-Entry Vehicles," PGAC, January 1964, pp. 90-98.

Considers designing a linear controller to compensate for atmospheric changes on control effectiveness in X-15 rocket plane.

[11] Horowitz, I. M., "Comparison of Linear Feedback Systems with Self-Oscillating Adaptive Systems," PGAC, October, 1964, pp. 386-392.

A "design" procedure is developed for an adaptive controller and applied to a plant. The resulting controller is compared with a linear controller for the same plant.

[12] Jacobs, O. L. R., "Two Uses of the Term 'Adaptive' in Automatic Control," PGAC, October 1964, pp. 574-575.

Draws a distinction between "adaptive" and "model-adaptive."

[13] Kaufman, H. and DeRusso, P. M., "An Adaptive Predictive Control System for Random Inputs," PGAC (Short Paper) October, 1964, pp. 540-545.

Utilizes a fast time predictive model to decide which polarity of a bang-bang control input should drive the plant. The model adjusts itself to better represent the plant.

[14] Knowles, J. B., "The Stability of a Proportional Rate Extremum Regulator," PGAC, July, 1964, pp. 256-264.

Considers noise. Mean square error criterion. Computes optimal gain setting from estimates of measurable mean square error. Stability investigated. Experimental results given. Assumes parabolic operating characteristic.

[15] Kozlov, O. M., "The Problem of Conditions for Identity of Systems that are Optimal with Respect to Different Criteria," A&RC, April, 1964, p. 1324.

The author studies conditions under which optimality of a system relative to one of the criteria in a certain class of quality criteria for automatic control systems entails optimality relative to all other criteria in the class.

[16] Krutova, I. N. and Rutkovskii, V. Yu., "Dynamics of a First-Order Adaptive system with a Model," A&RC, August, 1964, p. 175.

The influence of error algorithms, of the number of adjustable coefficients, and of their law of variation on the performance of an adaptive system with a model is considered. It is shown that the introduction of coefficient-variation adaption as a function only of the mismatch in the coordinates of the model and of the system and allows stabilization of the system when the self-aligning coefficients are negative.

[17] Krutova, I. N. and Rutkovskii, V. Yu., "Effects of Integrals in Laws Governing Variations of Modified Coefficients on the Dynamics of a Self-Adjusting System with a Model," A&RC, Nov., 1964, p. 441.

The effect of the integrals in the laws governing variations of the modified coefficients in the feedback loop (K_f) and the control-signal circuit (K_g) on the process of motion in a self-adjusting system is considered.

[18] Kumar, K.S.P. and Sridhar, R., "A Note on Combined Identification and Control," PGAC (Correspondence), January 1964, p. 118.

Introduction to Specific Optimal Control coupled with identification.

[19] Kumar, K.S.P. and Sridhar, R., "On the Identification of Control Systems by the Quasi-Linearization Method," PGAC, April, 1964, pp. 151-154.

Uses quasi-linearization technique to evaluate the coefficients of the assumed form of the plant differential equations.

[20] Kuntserich, V. M., "Investigation of One Class of High-Speed Adaptive Control Systems," A&RC, May, 1964, p. 1527.

The author describes a method of continuously varying the relative damping in both continuous and pulse systems. He obtains nonlinear differential and difference equations for adaptive systems with a method proposed for measuring quality criteria. He gives the results of simulating several types of adaptive systems.

[21] Kushner, H. J., "On the Optimum Timing of Observations for Linear Control Systems with Unknown Initial States," PGAC, April, 1964, pp. 144-150.

Shows that optimal timing of observations may be important. Done for a plant with linear dynamics, quadratic cost function, and no magnitude constraints.

[22] Leibovic, K. N., "The Principle of Contraction Mapping in Nonlinear and Adaptive Control Systems," JACC, 1964, p. 34 (also in PGAC, October, 1964, pp. 393-398).

In this paper the concept of contraction mapping is suggested as a means of controlling a plant. The paper only introduces this concept and does not indicate in any detail how one may design systems using it.

[23] Levin, M. J., "Estimation of a System Pulse Transfer Function in the Presence of Noise," PGAC, July, 1964, pp. 229-235.

Coefficients of a pulse transfer function are obtained as components of eigenvectors involved in an equation associated with cross-correlation of inputs and outputs. Noise is considered and certain variances are computed. Least squares estimates also discussed.

[24] Mendel, J. M., "On the Use of Orthogonal Exponentials in a Feedback Application," PGAC, July, 1964, pp. 310-312.

Extension of the identification scheme discussed in Mishkin and Braun using orthogonal exponentials.

[25] Miller, R. W. and Rob Roy, "Nonlinear Process Identification Using Decision Theory," PGAC, (Short Paper) Oct., 1964, pp. 538-540.

[26] Narendra, K. S. and McBride, L. E., "Multiparameter Self-Optimizing Systems Using Correlation Techniques," PGAC January, 1964, pp. 31-38.

Uses gradient in multidimensional parameter space. Orthogonalizes to get independent parameter variations. Results given for linear time invariant systems. No test signal or parameter perturbation are required. Cross correlation is used to obtain the gradients.

[27] Narendra, K. S. and Streeter, D. N., "An Adaptive Procedure for Controlling Undefined Linear Processes," PGAC October 1964, pp. 545-548.

An extension of paper by Narendra and McBride in PGAC, Jan., 1964. Cross correlation techniques utilized to obtain gradients with respect to controller parameter variations. Mean square error is performance criterion.

[28] Nelson, W. L., "On the Use of Optimization Theory for Practical Control System Design," PGAC, Oct., 1964, pp. 469-477.

Performance bounds relating various competing performance requirements of a system are evaluated using optimization techniques so that ultimate trade-offs achievable between these requirements are well understood. Using a specific example (Satellite Attitude Control) a performance surface is obtained relating fuel expenditure and control time to initial conditions. A controller is designed from this information.

[29] Pearson, A. E. and Sarachik, P. E., "On the Formulation of Optimal Control Problems," JACC, 1964, p. 13.

This paper presents a review of optimization literature emphasizing the essential similarity of formulations introduced therein. Included in the paper is a derivation of the Euler-Lagrange equations via the gradient of a given index of performance.

[30] Perlis, H. J., "The Utilization of Extremal Correlated Signals to Reduce the Self-Adaptive Cost Function," PGAC (Correspondence) Jan., 1964, p. 116.

[31] Perlis, H. J., "The Minimization of Measurement Error in a General Perturbation-Correlation Process Identification System," PGAC, Oct. 1964, pp. 339-345.

Using sinusoidal perturbation signals impulse response measurements obtained for linear time varying systems. Noise measurements included. Cross correlation techniques employed. When used in an adaptive loop identification not preferred. Instead an error is measured and minimized.

[32] Raevskii, S. Ya., "Statistical Method for Determining Essentially Nonlinear Characteristics of Plants Under Control," A&RC, Nov., 1964, pp. 814-820.

A method is proposed for determining the essentially nonlinear characteristics of controlled plants. The method is based on general results in statistical dynamics. Some aspects of the method of practical importance are considered.

[33] Sugie, N., "An Extension of Fibonaccian Searching to Multi-dimensional Cases," PGAC, January, 1964, p. 105.

Self-explanatory title. One problem in the extension is that the number of points to be searched goes up quite fast.

[34] Tyler, J. S., "The Characteristics of Model-Following Systems as Synthesized by Optimal Control," JACC, 1964, p. 40 (also PGAC, October, 1964, pp. 485-493).

This paper presents extensions of Kalman's work on the use of models in optimal control. It is pointed out that the use of a model extends the useful range of quadratic performance indices for linear systems.

[35] Zaborszky, J. and Humphrey, W. L., "Control Without Model or Plant Identification," JACC, 1964, p. 366.

In this paper, the problem of controling an unknown, nonlinear time-varying plant is considered. A model of the plant, based upon measurements, is obtained using a volterra series. Control is based upon the current response of the plant and on the "current sensitivity" of the plant to input disturbances.

[36] Belenkii, A. A. and Chelyutkin, A. B., "The Dynamics of a Continuous Automatic Optimizer for One Class of Systems," A&RC, November, 1963, pp. 720-734.

Dynamics of continuous, automatic optimizers in perturbation-controlled systems are considered for systems with automatic adjustment of parameters. Transient responses and stability conditions, under statistical as well as deterministic perturbations are analyzed. A pseudo cross correlation function to remove influence of noise.

[37] Bobrov, Yu. I.; Kornilov, R. V.; and Putsillo, V. P., "Determination of the Optimizer Control Law by Taking into Account the Relaxation of the System to be Controlled," A&RC, Sept., 1963, pp. 175-182.

The structure of the optimizer's controlling component is determined by analyzing the system's motion in the phase plane. A class of optimizing control relay systems where the system to be controlled is represented by a first-order factor and a nonlinear element whose characteristic has a single extremum is considered.

[38] Bozhukov, V. M. and Kukhtenko, V. I., "A Method of Design of Adaptive, Automatic-Control Systems with the Stabilization of Frequency Characteristics," A&RC, December, 1963, p. 869.

A system is developed which will determine its own frequency characteristics. By adjusting gains the system will keep these characteristics constant in spite of variable plant parameters.

[39] Chatterjee and Bhattacharyya, "Measurement of an Impulse Response of a System with a Random Input," PGAC, April, 1963, pp. 186-187.

An implementation of a binary noise cross correlation technique.

[40] Chestnut, H.; Duersch, R. R. and Hahn, G. J., "Automatic Optimizing of a Poorly Defined Process," JACC, 1963, p. 54.

A straightforward statistical analysis is applied to a gradient technique. The effect of each step in the search is analyzed to see if the correct choice has been made. Using a model and assuming a pulse input, transient data is extrapolated to steady state.

[41] Elkind, J. I.; Green, D. M. and Starr, E. A., "Application of Multiple Regression Analysis to Identification of Time Varying Linear Dynamic Systems," PGAC, April, 1963, pp. 163-166.

System input is applied to model comprised of many filters, orthogonal to each other, and in parallel. Regression techniques used to add the outputs to obtain the impulse response. For time varying case the above is done for "short" periods of time. Noise is included.

[42] Eveleigh, V. W., "General Stability Analysis of Sinusoidal Perturbation Extrema Searching Adaptive Systems," JACC, 1963, p. 91.

This short note, referring to an unpublished paper, is concerned with checking sinusoidal perturbation frequencies with describing functions.

[43] Eykhoff, P., "Some Fundamental Aspects of Process-Parameter Estimation," PGAC, October, 1963, pp. 347-357.

A general unifying discussion of various techniques, indices of performance, etc. for estimating the coefficients of a process. Several techniques are discussed in detail.

[44] Gibson, J. E., Nonlinear Automatic Control, McGraw-Hill Book Company, New York, 1963.

Chapter 11 of this text contains discussions of a number of aspects of the adaptive control problem. Self-optimizing systems are first defined followed by a presentation of several identification techniques. Treatments of various adaptive schemes follow. The point of view that an adaptive control system consists of three phases: Identification, modification and decision is presented.

[45] Gieseking, D., "An Optimum Bistable Controller for Increased Missile Autopilot Performance," PGAC, Oct., 1963, pp. 306-309.

A general form for a closed loop controller is obtained for a quadratic integral performance index. The controller parameters, which depend on plant parameters, vary as the plant parameters vary. Analog results are presented.

[46] Giloi, W., "Optimized Feedback Control of Dead Time Plants by Complementary Feedback," JACC, 1963, p. 211.

This paper presents techniques for designing controllers for linear plants with large dead time. The basic structure used in the technique is an approximate analog simulation of the plant. The dead time of the simulation is adjusted to keep it in correspondence with the dead time of the plant.

[47] Harris, R. J., "Trajectory Simulation Applicable to Stability and Control Studies of Large Multi-Engine Vehicles," NASA-TN-D-1838, August, 1963.

Three-dimensional, six-degree-of-freedom trajectory simulation is formulated. Slosh and elasticity ignored. A numerical example is included. Results are compared, where possible, with a two-dimensional simulation.

[48] Ho, Y. and Whalen, B. H., "An Approach to the Identification and Control of Linear Dynamic Systems with Unknown Parameters," PGAC, July, 1963, pp. 255-256.

Identification is performed using an estimation of the states.

of neglected parameters (i.e., in plant but not in model). Discusses stochastic approximation. Plant time variations are taken into account directly.

[55] Lindenlaub, J. C. and Cooper, G. R., "Noise Limitations of System Identification Techniques," PGAC, January, 1963, pp. 43-48.

Discussion of system identification using binary noise and cross correlation. Impulse response is the result of the identification.

[56] Lubbock, J. K. and Barker, H. A., "A Solution of the Identification Problem", JACC, 1963, p. 191.

Identification is achieved using orthogonal functions operating on normal plant disturbances. Perturbation signals or stochastic inputs are not required.

[57] Mósner, P., "A Perturbation Approach to an Adaptive Sampled Data Control System," PGAC, April, 1963, pp. 171-172.

A digitally controlled sampled data system is monitored at output and resulting changes in plant parameters are determined. The controller is then altered to cancel the effects of these variations.

[58] Osovskii, L. M., "Linear Self-Adjusting Simulators with Adjustments with Respect to the Phase Response," A&RC, September, 1963, pp. 165-174.

The paper considers a model-adaptive system. A second-order example is presented.

[59] Osovskii, L. M., "On a Class of Nonlinear Self-Adjusting Simulators with Adjustment with Respect to Phase and Amplitude Response," A&RC, October, 1963, pp. 341-353.

This paper considers a model-adaptive system.

[60] Pearson, A. E., "On Adaptive Optimal Control of Nonlinear Processes," JACC, 1963, p. 80. (Also J.B.E., March, 1964, pp. 151-160)

This paper is concerned with identification of an unknown plant and with the implementation of a controller based upon this identification. Functional analysis is employed to obtain an expression for the gradient of an assumed index of performance in terms of a measurable differential associated with the plant. Optimization is achieved by applying a sequence of inputs to the plant which force the gradient to zero.

[61] Pottle, C., "The Digital Adaptive Control of a Linear Process Modulated by Random Noise," PGAC, July, 1963, pp. 228-234.

Plant identification is performed using state variable estimation and correlation techniques are used to predict future plant behavior. The optimal controller minimizes the predicted plant over the near future. No input required if plant statistics are known.

[62] Puri, N. N. and Weygandt, C. N., "Transfer Function Tracking of a Linear Time Varying System by Means of Auxiliary Simple Lag Systems," JACC, 1963, p. 200.

Linear, quasi-stationary, noise-free systems are considered.

[63] Rajaraman, V. and Wentz, H. J., "On Stability and Steepest Descent," PGAC, (Correspondence) January, 1963 pp. 61-62.

Self-explanatory title.

[64] Rob Roy, "Predictive Delay Line Synthesizer," PGAC, April, 1963, pp. 185-186.

Approximates convolution by summation. Examines input and output records to get impulse response. Considers effects of noise. Uses a delay line to aid in this problem.

[65] Roy, R., Miller, R. W., and DeRusso, "An Adaptive Predictive Model for Nonlinear Processes with Two-Level Inputs," JACC, 1963, p. 204.

Identification time is on the order of 500 to 10,000 time constants.

[66] Smyth, R. K. and Nahi, N. E., "Phase and Amplitude Sinusoidal Dither Adaptive Control System," PGAC, October 1963, pp. 311-320.

Dither frequency used to detect changes in plant characteristics. Two control loops used: one to adjust loop gain, the other to adjust loop phase. Results given for some examples.

[67] Waymeyer, W. K. and Sporing, R. W., " Closed Loop Adaptation Applied to Missile Control," PGAC, April, 1963, pp. 157-160.

Feedback gain self-adjusted to maintain closed loop poles at a certain desired position in the s-plane. Simulation results given.

[68] Bellman, R. E. and Dreyfus, S. E., Applied Dynamic Programming, Princeton University Press, Princeton, New Jersey, 1962.

This text covers a wide range of topics involving optimization via dynamic programming. Computational techniques such as quasilinearization, and the gradient method are discussed.

[69] Bernard, J. W. and Lefkovitz, I., "An Approach to Optimizing Control Based on a Generalized Dynamic Model," JACC, 1962.

This paper is on model Adaptive Control. The specific model studied is a chemical reactor. It is assumed that a steady state mathematical model of the system and the control for static optimization are known. An adaptive controller is designed to handle the transient effects in the system.

[70] Bishop, K. A. and Sliepcevich, C. M., "Techniques for Identification of Linear and Linear Time-Varying Processes," JACC, 1962.

This paper discusses a method of obtaining the differential equations of a Linear system from the system impulse response. The system may have time-varying coefficients, but these coefficients must be expandable in a Taylor Series.

[71] Ehrich, F. F., "Analysis of a Bivariant Optimizing Control," J.B.E., September 1962, pp. 410-411.

Two parameters are varied to obtain a minimum via a gradient. A specific example is included using resolvers.

[72] Elkind, J. I.; Green, D. M.; and Starr, E. A., "Application of Multiple Regression Analysis to Identification of Time-Varying Linear Dynamic Systems," JACC, 1962.

In this technique a test signal (noise) is applied to a linear system and the resulting inputs and outputs are sampled. These measurements are used to determine the coefficient of a set of orthonormal filters. This set of filters is used to approximate the system transfer function. For good operation, the approximate location of the poles of the plant transfer function must be known.

[73] Frait, J. S. and Eckman, D. P., "Optimizing Control of Single Input Extremum Systems," JBE, March,1962, pp. 85-90. (Also JACC, 1961.)

The performance index, an algebraic function of the single output, is extremized by applying appropriate inputs using a device called a "divider optimizer." Experimental results are given. The plant dynamics are assumed known.

[74] Friedland, B., "The Structure of Optimum Control Systems," JBE, March, 1962, pp. 1-11. (Also JACC, 1961)

An optimal controller as dictated by the Maximum Principle is implemented. The solution of the adjoint equations is performed on line (assumes knowledge of initial conditions of adjoint variables). The form of the plant must be known.

[75] Hsu, J. C. and Meserve, W. E., "Decision-Making in Adaptive Control Systems," PGAC, Jan., 1962, pp. 24-32.

This article considers decision making in systems where the system parameters are not known exactly and their measurement or identification is obscured by noise.

[76] Krosovski, A. A., "Optimal Methods of Search in Continuous and Pulsed Extremum Control Systems," Proc. 1st International Symposium on Optimizing and Adaptive Control, 1962, p. 19.

This paper is concerned with finding the components of a gradient in a noisy system. A mean squared error criteria is used, resulting in an optimal equation for finding the gradient.

[77] Moe, M. L. and Murphy, G. J., "An Approach to Self-Adaptive Control Based on the Use of the Time Moment and a Model Reference," JACC, 1962.

[78] Nomoto, A., "Dynamic Programming for Direct Optimization Systems," Proc. 1st International Symposium on Optimizing and Adaptive Control, 1962, p. 39

This paper is concerned with the application of dynamic programming to the optimization of a discrete system.

[79] Pospelov, G. S., "On the Principle of Design of Certain Types of Adaptive Control Systems," Proc. 1st International Symposium on Optimizing and Adaptive Control, 1962, p. 147.

This is a general paper which discusses optimization of discrete systems with slowly varying parameters via the Maximum Principle.

[80] Puri, N. N. and Weyganat, C. N., "Multivariable Adaptive Control System," JACC, 1962.

[81] Smyth, R. K. and Davis, J. C., "A Self-Adaptive Control System for a Space Booster of the Saturn Class," JACC, 1962.

Presents a tracking notch filter for decoupling of the flexible modes of a space booster from the rigid body mode.

[82] Whitaker, H. P., "Design Capabilities of Model Reference Adaptive Systems," R-374 MIT Inst. Lab., July, 1962.

A review of the results achieved in the development of model-reference adaptive control, and description of performance capabilities, design procedures, and applications.

[83] Bellman, R. E., Adaptive Control Processes, A Guided Tour, Princeton University Press, Princeton, New Jersey, 1961.

This book contains a number of possible applications of dynamic programming. Emphasis is on optimization problems rather than on adaptive control systems.

[91] McGrath, R. J.; Rajaraman, V. and Rideout, V. C., " A Parameter-Perturbation Adaptive Control System," PGAC, May, 1961, pp. 154-162.

This paper considers a model adaptive system with multiple loops, each one of which is perturbed by a different test signal. Knowledge of the form of the plant transfer function is required.

[92] Meditch, J. S., "A Class of Predictive Adaptive Controls," Ph.D. Thesis, Purdue University, 1961.

This thesis describes a control system which optimizes an integral-squared performance index on a per interval basis. The contr l signal is assumed to be a linear combination of known orthonormal time functions. Optimization is achieved by picking the coefficients of this linear combination so as to minimize the performance index. Optimization is performed on line.

[93] Mishkin, E. and Braun, L., Adaptive Control Systems, McGraw-Hill Book Company, New York, 1961.

This book, edited by Mishkin and Braun, contains a variety of specialized topics associated with adaptive control systems. Chapter 1 contains a general discussion of the Adaptive Control process. In Chapter 3, the identification problem is considered. Several specific adaptive control systems are discussed in Chapter 10. Chapter 11 includes some adaptive processes employing the digital computer.

[94] Qvarnstrom, B., "Transfer Function Determination in the Presence of Noise for a Set of Significant Input Functions," Instruments and Measurement, Proc. 5th International Instruments and Measurement Conference, Stockholm, 1960, pp. 56-71, 1961.

[95] Schultz, W. C. and Rideout, V. C., "Control System Performance Measures: Past, Present, and Future," PGAC, Feb., 1961, pp. 22-35.

Discussion of general integral criteria.

[96] Truxal, J. G., "Computers in Automatic Control Systems," Proc. IRE, 49, 1961, pp. 305-312.

Review type article to establish the state of the art at that time.

[97] Weygandt, C. N. and Puri, N. N., "Transfer-Function Tracking and Adaptive Control Systems," PGAC, May, 1961, pp. 162-166.

The system automatically tracks parameters in the denominator polynomial of the plant transfer function. This is achieved using a number of perturbating signals, each one of different frequency than the others.

[98] Zaborszky, J., Luedde, W. J., and Wendl, M. J., "New Flight Control Techniques for a Highly Elastic Booster," ASD-TR-61-231, Sept., 1961.

Uses a Digital Adaptive Filter to separate rigid body and flexible body signals even when frequencies are very close together. Discriminate by difference in damping.

[99] Aseltine, J. A. and Anderson, G. W., "A Study to Determine the Feasibility of a Self Optimizing Automatic Flight Control System," WADD-TR-60-201, June 1960.

[100] Braun, L.; Mishkin, E. and Truxal, J., "Approximate Identification of Process Dynamics in Computer Controlled Adaptive Systems," IFAC, 1960, pp. 596-603.

This technique uses orthonormal functions to identify certain parameters associated with the dynamics of linear systems.

[101] Cooper, G. R., Gibson, J. E., et.al., "Philosophy and State of the Art of Adaptive Systems," TR No. 1, AF 33(616)-6890, Purdue University, July, 1960.

This report contains a general introduction to the adaptive control concept. It is a broad literature review which might serve as a bibliography.

[102] Eckman, D. P. and Lefkowitz, I., "Principle of Model Techniques in Optimizing Control," IFAC, 1960, p. 970.

This paper is primarily concerned with methods for obtaining an optimum controller for a given physical plant.

[103] Fleischner, P. E., "Optimum Design of Passive-Adaptive Systems with Varying Plants," Technical Report 400-16, Dept. of E.E., New York University, 1960.

This report discusses a method for specifying the optimum overall transfer function and sensitivity for a given system. The difference in outputs of the actual plant and a model is used as an input to the controller. The control signal is chosen so as to minimize the expected value of a mean squared error.

[104] Geissler, E. D., "Problems in Attitude Stabilization of Large Guided Missiles," Aerospace Engineering, October, 1960, p. 24.

A general exposition of the problems of artificially stabilizing large guided missiles.

[105] Gibson, J. E., "Adaptive and Self-Optimalizing Systems," IFAC, 1960, p. 586.

This article is primarily concerned with defining Adaptive Control Systems. Some of the basic problems one might encounter in designing such systems are discussed. Comments are given on the application of gradient methods to self-optimizing systems.

[112] Freimer, M., "A Dynamic Programming Approach to Adaptive Control Processes," PGAC, Nov., 1959, p. 10.

This paper considers a performance index which is a function of some stochastic disturbances. Dynamic programming is used to converge on the true statistical properties of the disturbances and to pick a control variable to minimize the performance index. A stochastic disturbance in the sense used here, might be a statistically defined target location which the plant trajectory is supposed to hit.

[113] Gibson, J. E. and McVey, E. S., "Multidimensional Adaptive Control," NEC, 1959, p. 12.

Parameter perturbations and self adjustment to minimize an index of performance. Experimental results are given.

[114] Gregory, P. C., (Editor), "Proc. of Symposium on Self Adaptive Automatic Flight Control Systems," ARDC, WADC-TR-59-49, 1959.

[115] Leonav, Y. P. and Liapatov, L. N., "The Use of Statistical Methods for Determining the Characteristics of Objects. (A Survey)," A&RC, Sept., 1959, pp. 1254-1268.

Several techniques associated with the identification of a plant transfer function are discussed. Amongst these are methods for computing amplitude and phase characteristic. A method for determining system weighting functions via correlation techniques is also presented.

[116] Margolis, M. and Leondes, C. T., "On the Philosophy of Adaptive Control for Plant Adaptive Systems," NEC, 1959, p. 27.

A section is included discussing the operation of the adjusting mechanism in an adaptive control.

[117] Margolis, M. and Leondes, C. T., "A Parameter Tracking Servo for Adaptive Control Systems," PGAC, Nov., 1959, pp. 100-111.

This paper deals with the problem of identifying unknown coefficients of a system differential equation. A model is used, whose differential equation is of the same form as that of the plant. The same input signal that is applied to the plant is also applied to the model. The coefficients of the model are adjusted so as to bring the output of the model into correspondence with the output of the plant.

[118] Stromer, P. R., "Adaptive or Self Optimizing Control Systems - A Bibliography," PGAC, May, 1959, p. 65.

This is an annotated bibliography of adaptive control papers written before 1959.

[125] Henderson, J. S. and Pengilley, C. J., "The Experimental Determination of System Transfer Functions from Normal Operating Data," Journ. Brit. IRE, March, 1958, pp. 179-186.

A cross correlation technique.

[126] Kalman, R., "Design of a Self-Optimizing Control System," Trans. ASME, 80, Feb., 1958, pp. 468-478.

Employs a digital computer as a controller. Noisy measurements allowed.

[127] Kalman, R. E. and Koepcke, R. W., "Optimal Synthesis of Linear Sampling Control Systems Using Generalized Performance Indices," Trans. ASME, 80, Nov., 1958, pp. 1820-1826.

This paper presents a simple adaptation of dynamic programming to the design of optimal controllers for linear plants. The paper deals with sampled data systems, but extensions to continuous systems are claimed to be straightforward.

[128] Stakhovskii, R. I., "Twin Channel Automatic Optimalizer," A&RC, August, 1958, pp. 729-740.

This is a report on the results of experimentation with a two parameter gradient search. Equipment was constructed and is discussed in some detail.

[129] Woodrow, R. A., "Closed-Loop Dynamics from Normal Operating Records," Trans. Soc. Instrum. Technol., Sept. 1958, pp. 101-105.

[130] Cowley, P. E. A., "The Application of an Analog Computer to the Measurement of Process Dynamics," Trans. ASME, 79, 1957, pp. 823-832.

Plant identification is achieved by applying sine and cosine test signals to the plant input. The analysis assumes that the test signals will disturb the plant. The system must be operating open-loop when the identification is performed.

[131] Drenick, R. F. and Shahbender, R. A., "Adaptive Servomechanisms," Trans. AIEE, pt. II, 1957, pp. 286-291.

Input or signal adaptive system. The system as presented is impractical.

[132] Goodman, T. P., "Determination of the Characteristics of Multi-input and Nonlinear Systems from Normal Operating Records," Trans. ASME, 79, 1957, pp. 567-575.

A plant identification method is presented in which the plant impulse response h(T) is broken up into small rectangles of width T over which h(T) is assumed to be constant. The plant output autocorrelation function is used in conjunction with the input-output cross correlation function to calculate the constant numbers h_i which describe h(T).

[133] Jensen, J. R., "Notes on Measurement of Dynamic Characteristics of Linear Systems," Servoteknisk Forskningslaboratorium, Danmarks tekniske hoyskole, 1957-1959.

[134] Turin, G. L., "On the Estimation in the Presnece of Noise of Impulse Response of a Random Linear Filter," IRE, Trans. in Information Theory, March, 1957, pp. 5-10.

[135] Chang, C. M., Goodman, T. P., and Reswick, J. B., "Use of Correlation Functions to Determine System Characteristics Without Applying Artificial Disturbances," Regelungstecknik, Tagung Geidelberg, 1956, pp. 251-256.

[136] Goodman, T. P. and Reswick, J. B., "Determination of System Characteristics from Normal Operating Records," Trans. ASME, 78, 1956, pp. 259-271.

This paper discusses identification of a linear system via cross correlation techniques. White noise is required for good results.

[137] Zadeh, L. A., "On the Identification Problem," IRE, PGCT, Dec., 1956.

[138]. Lang, G. and Ham, J. M., "Conditional Feedback Systems - A New Approach to Feedback Control," Trans. of the AIEE, Pt. II, 1955, pp. 152-161.

An approach is presented which permits requirements on input-output response and on disturbance-output response to be met independently for non-linear systems. The method is basically a feedback method where the input signal, rather than a controller parameter, is modified to achieve the desired result.

[139] Margolis, S. G., "Measurement of Industrial Process Behaviour," M.I.T. Res. Lab. of Electronics Quart. Progr. Repts., April, 1955, pp. 42-46; July, 1955, pp. 49-52.

These reports deal with the experimental determination of transfer functions by correlation techniques. Discrete auto-correlation functions are used.

[140] Moore, E. F., "Gedanken-Experiments on Sequential Machines;" Automata Studies, Princeton University Press, Princeton, New Jersey, 1954, pp. 129-156.

This paper discusses identification experiments for finite, deterministic, automata.

[141] Draper, C. S. and Li, Y. T., "Principles of Optimalizing Control Systems and an Application to an I. C. Engine," ASME Publication, New York, 1951.

In this paper, the problem of climbing a unimodal hill is considered. The problem is attacked using both gradient methods and sinusoidal perturbation techniques.

APPENDIX B

EQUATIONS OF MOTION FOR THE MODEL VEHICLE

The rigid body and flexible body equations for the model vehicle are derived below. Included also is an expression for the bending moment at station x. [1,2,3,4,5] are general references on the subject. Figure B.1 defines the pertinent symbols.

Pitch plane rigid body equations are derived as follows. Referring to Figure B.2, write the expressions for kinetic and potential energy

$$\begin{aligned} T &= \frac{1}{2} m(\dot{u}^2 + \dot{v}^2) + \frac{1}{2} I_{xx}(\dot{X} + \dot{\phi}_R)^2 \\ V &= mgv \end{aligned} \tag{B.1}$$

The generalized forces for coordinates u, v, and ϕ_R are

$$\begin{aligned} Q_u &= (\frac{F}{2} - X) \sin (X + \phi_R) + \frac{F}{2} \sin (X + \phi_R + \beta) + N \cos (X + \phi_R) \\ Q_v &= (\frac{F}{2} - X) \cos (X + \phi_R) + \frac{F}{2} \cos (X + \phi_R + \beta) - N \sin (X + \phi_R) \\ Q_{\phi_R} &= (X_{CP} - X_{CG}) N - (X_{CG} - X_\beta) \frac{F}{2} \sin \beta \end{aligned} \tag{B.2}$$

For these coordinates the equations of motion are

$$\begin{aligned} m\ddot{u} &= Q_u \\ m\ddot{v} + mg &= Q_v \\ I_{xx}(\ddot{X} + \ddot{\phi}_R) &= Q_{\phi_R} \end{aligned} \tag{B.3}$$

Substitute into equations (B.3) the geometric relationships

$$\begin{aligned} u &= u_o + Z \cos X + Y \sin X \\ v &= v_o - Z \sin X + Y \cos X \end{aligned} \tag{B.4}$$

Figure B.1

Definition of Symbols

Φ_R	attitude angle of the rigid body
$\Phi(x)$	attitude angle at station x
α	angle-of-attack
α_T	angle-of-attack measured by angle of attack sensor
β_R	control deflection angle
Z	direction normal to reference
Y	direction parallel to reference
z	direction normal to vehicle centerline
x	distance along vehicle centerline from vehicle base
$m(x)$	mass per unit length of vehicle
m	total vehicle mass
I_{xx}	pitch plane moment of inertia about CG
V	inertial velocity of vehicle
V_{REL}	velocity relative to wind
V_w	wind velocity
α_w	angle between relative and inertial velocity vectors
υ	angle between inertial velocity vector and reference axis
X	angle between vertical at launch and desired reference axis
F	total thrust
X	drag force (longitudinal aerodynamic force)
N	normal aerodynamic force
N'	aerodynamic force
R'	thrust of control engines
X_{CG}	center of gravity
X_{CP}	center of pressure

Figure B.1

Definition of Symbols (Continued)

X_β	gimbal position
$BM(x)$	bending moment at station x
q	aerodynamic pressure
A	reference cross-sectional area
$C_{Z\alpha}$	normal aerodynamic force coefficient
$\frac{\partial C_{Z\alpha}}{\partial X}(x)$	normal aerodynamic force per unit length
$EI(x)$	bending stiffness at x
$y_i(t, x) = Y_i(x)\ \eta_i(t)$	deflection normal to vehicle centerline due to i^{th} mode
$Y_i(x)$	normalized natural mode shapes
$\eta_i(t)$	normal coordinates
ω_i	bending frequency of the i^{th} mode
ξ_i	damping of the i^{th} bending mode
Q_i	generalized force of the i^{th} bending mode
M_i	generalized mass of the i^{th} bending mode
I_E	engine moment of inertia about gimbal point
S_E	first moment of swivel about gimbal point
M_E	mass of swiveled engine

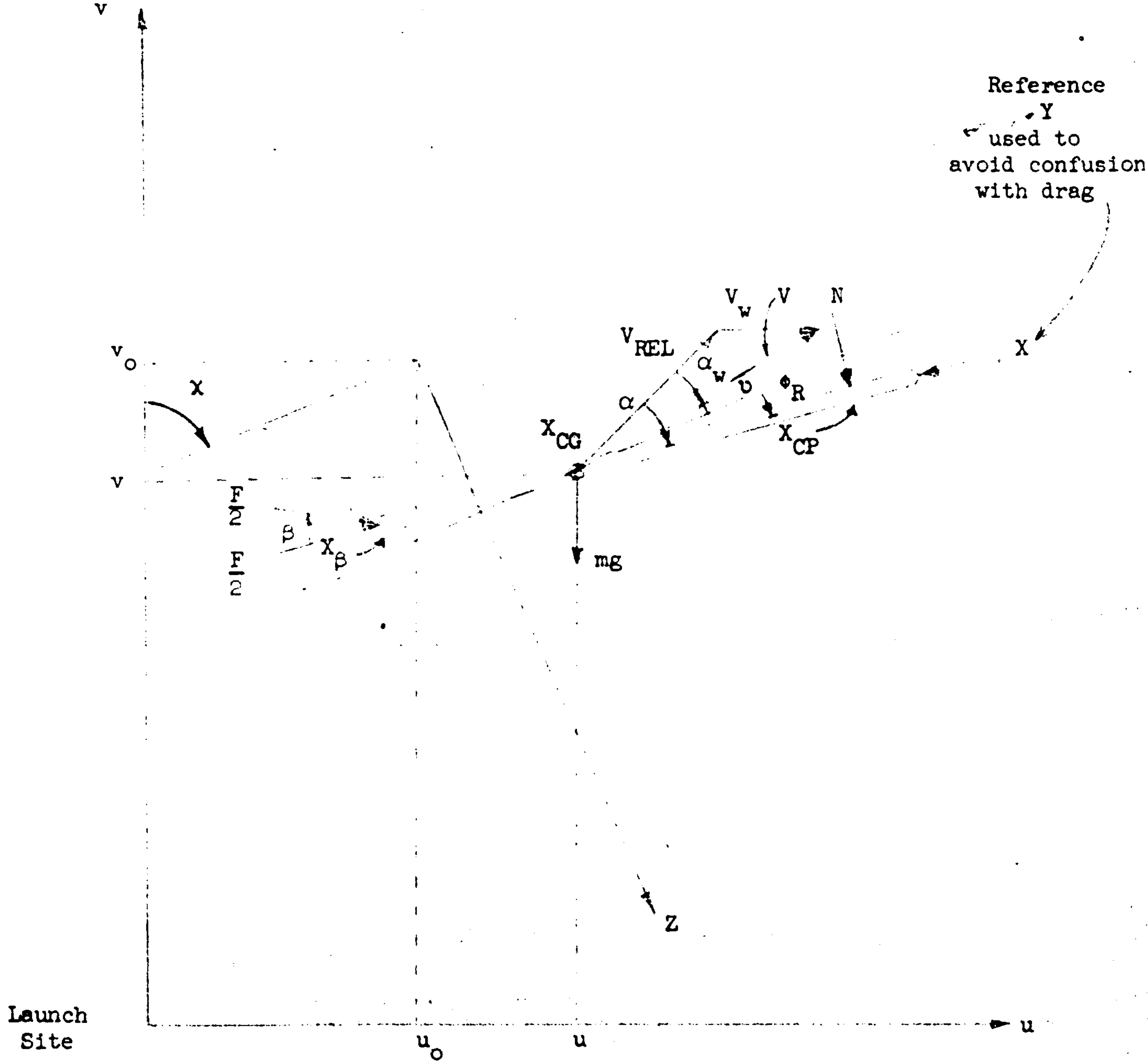


Figure B.2

Rigid Body Coordinate System - Pitch Plane and assume $\dot{\chi} \cong \ddot{\chi} \cong 0$. Make small angle assumptions for ϕ_R, β, and α.

$$\begin{aligned} \cos(\chi + \phi_R) &\cong \cos\chi - \phi_R \sin\chi \\ \sin(\chi + \phi_R) &\cong \sin\chi + \phi_R \cos\chi \\ N &\cong N'\alpha \\ \frac{F}{2}\sin\beta &\cong \frac{F}{2}\beta = R'\beta \end{aligned} \tag{B.5}$$

In order to separate the guidance and control equations, let

$$\beta = \beta_G + \beta_R = \text{guidance} + \text{control} \tag{B.6}$$

Substituting equations (B.4), (B.5) and (B.6) into (B.3) yields

$$\begin{aligned} m\ddot{u}_o + m\ddot{Z}\cos\chi + m\ddot{Y}\sin\chi &= [(F - X) - N'\alpha\,\phi_R]\sin\chi \\ &\quad + [(F - X)\phi_R + R'(\beta_G + \beta_R) + N'\alpha]\cos\chi \\ m\ddot{v}_o - m\ddot{Z}\sin\chi + m\ddot{Y}\cos\chi + mg &= [(F - X) - N'\alpha\phi_R]\cos\chi \\ &\quad - [(F - X)\phi_R + R'(\beta_G + \beta_R) + N'\alpha]\sin\chi \\ I_{xx}(\ddot{\chi} + \ddot{\phi}_R) &= (X_{CP} - X_{CG})N'\alpha - (X_{CG} - X_\beta)R'(\beta_G + \beta_R) \end{aligned} \tag{B.7}$$

The equations governing the nominal trajectory (gravity turn) are found by assuming the following conditions hold on the vehicle equations of equations (B.7).

$$\begin{aligned} \alpha &= \beta_R = 0 \\ \phi_R &= \dot{\phi}_R = \ddot{\phi}_R = 0 \\ Z &= \dot{Z} = \ddot{Z} = 0 \\ Y &= \dot{Y} = \ddot{Y} = 0 \end{aligned} \tag{B.8}$$

The resulting <u>Guidance Equations</u> are

$$\begin{aligned} m\ddot{u}_o - R'\beta_G\cos\chi - (F - X)\sin\chi &= 0 \\ mg + m\ddot{v}_o + R'\beta_G\sin\chi - (F - X)\cos\chi &= 0 \\ I_{xx}\ddot{\chi} + \ell_\beta R'\beta_G &= 0 \end{aligned} \tag{B.9}$$

where $\ell_B = X_{CG} - X_\beta$.

Substitution of equations (B.9) into (B.7) and grouping terms yields the following two equations

$$[m\ddot{Z} - (F - X)\phi_R - R'\beta_R - N'\alpha] \cos \chi + [m\ddot{Y} - (F - X) + N'\alpha\phi_R] \sin \chi = 0 \tag{B.10}$$

$$I_{xx}\ddot{\phi}_R = -(X_{CG} - X_{CP})N'\alpha - (X_{CG} - X_{\beta})R'\beta_R$$

Neglecting $\alpha\phi_R$ and requiring equations (B.10) to hold for all χ yields the Control Equations

$$\begin{aligned} \ddot{\phi}_R &= - C_1\alpha - C_2\beta_R \\ \ddot{Z} &= C_3\alpha + C_4\beta_R + C_5\phi_R \\ \ddot{Y} &= C_5 \end{aligned} \tag{B.11}$$

where the coefficients C_i are defined below and plotted as functions of time into flight in Figures B.3 through B.8.

$$\begin{aligned} C_1 &= \frac{N'}{I_{xx}} (X_{CG} - X_{CP}) \\ C_2 &= \frac{R'}{I_{xx}} (X_{CG} - X_{CP}) \\ C_3 &= \frac{N'}{m} \\ C_4 &= \frac{R'}{m} \\ C_5 &= \frac{F-X}{m} \\ C_7 &= \frac{1}{V} \end{aligned} \tag{B.12}$$

The angular relationship necessary to complete the representation of the rigid body model in equations (B.11) is

$$\alpha = \phi_R - \upsilon + \alpha_w \tag{B.13}$$

where the small angle assumptions yield

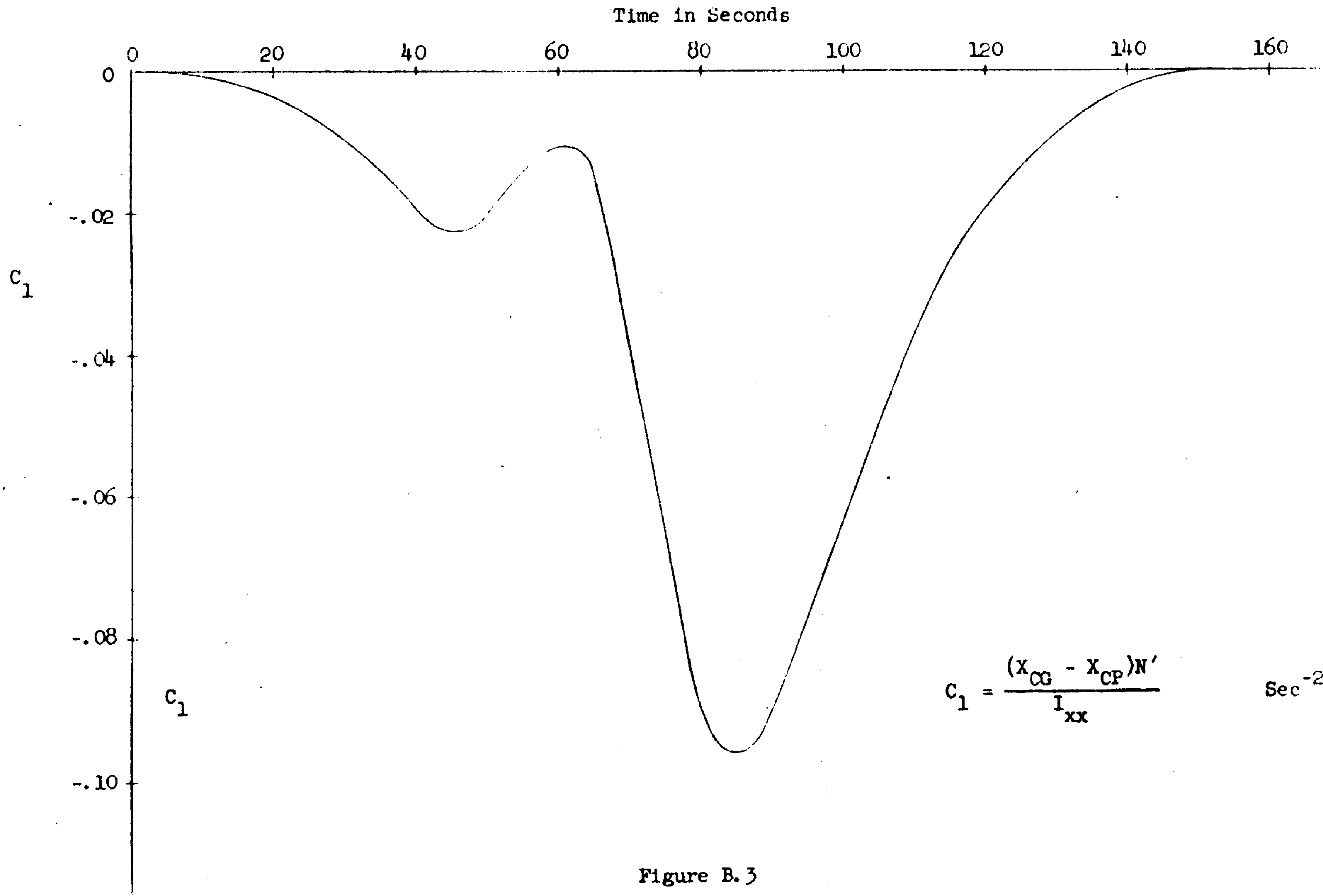


Figure B.3

Variation of C_1 with Time

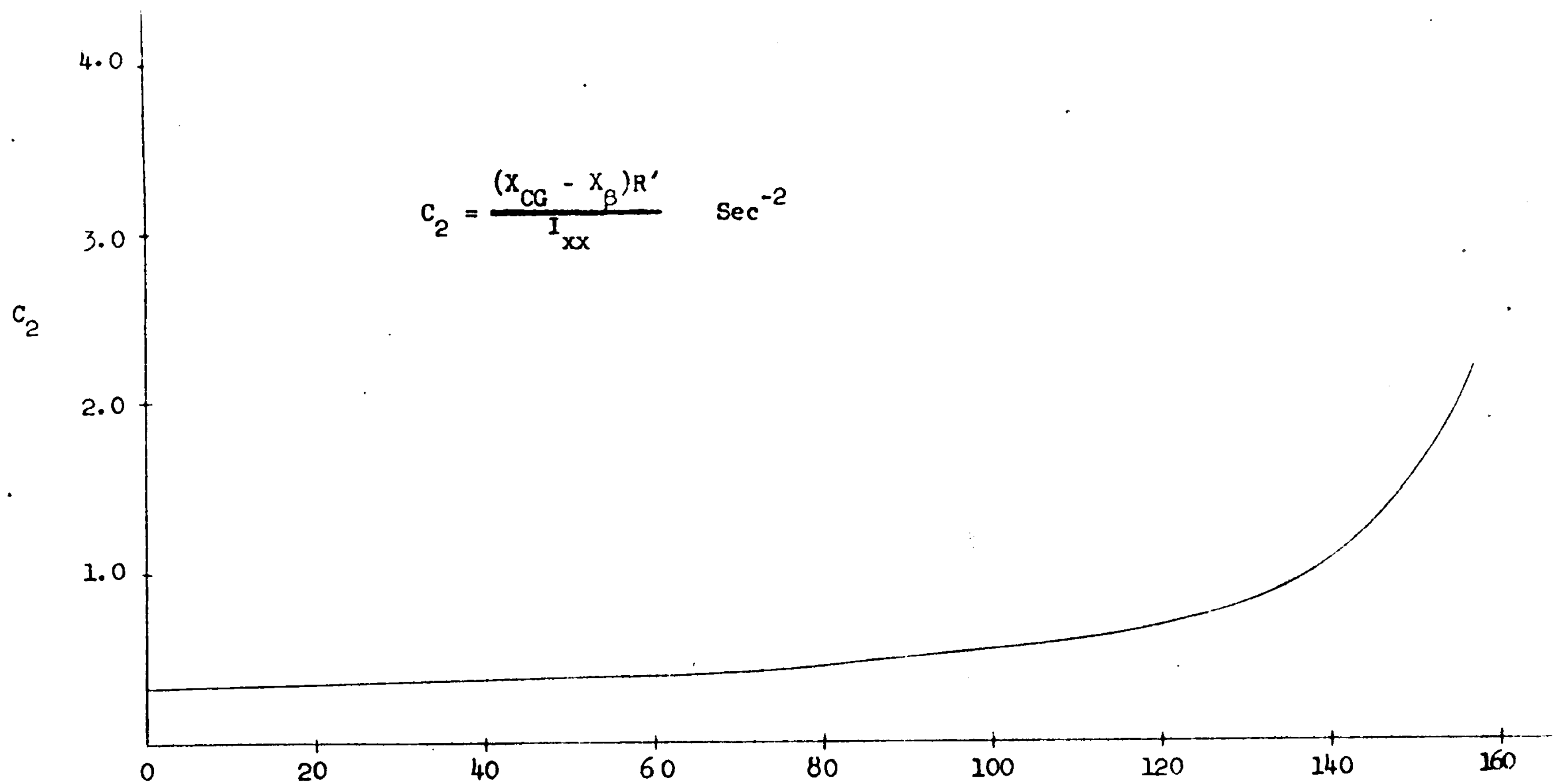


Figure B.4

Variation of C_2 with Time

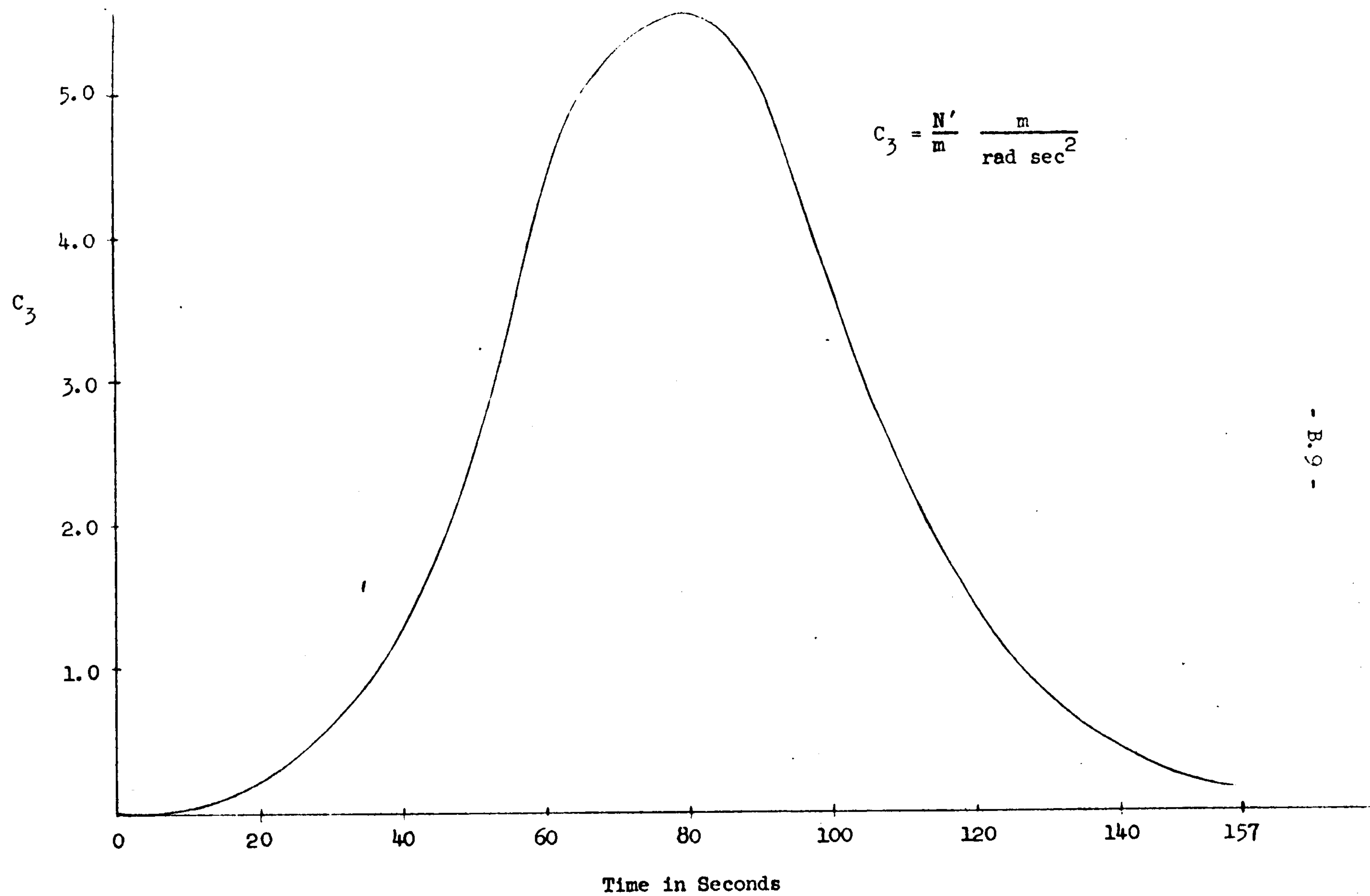


Figure B.5

Variation of C_3 with Time

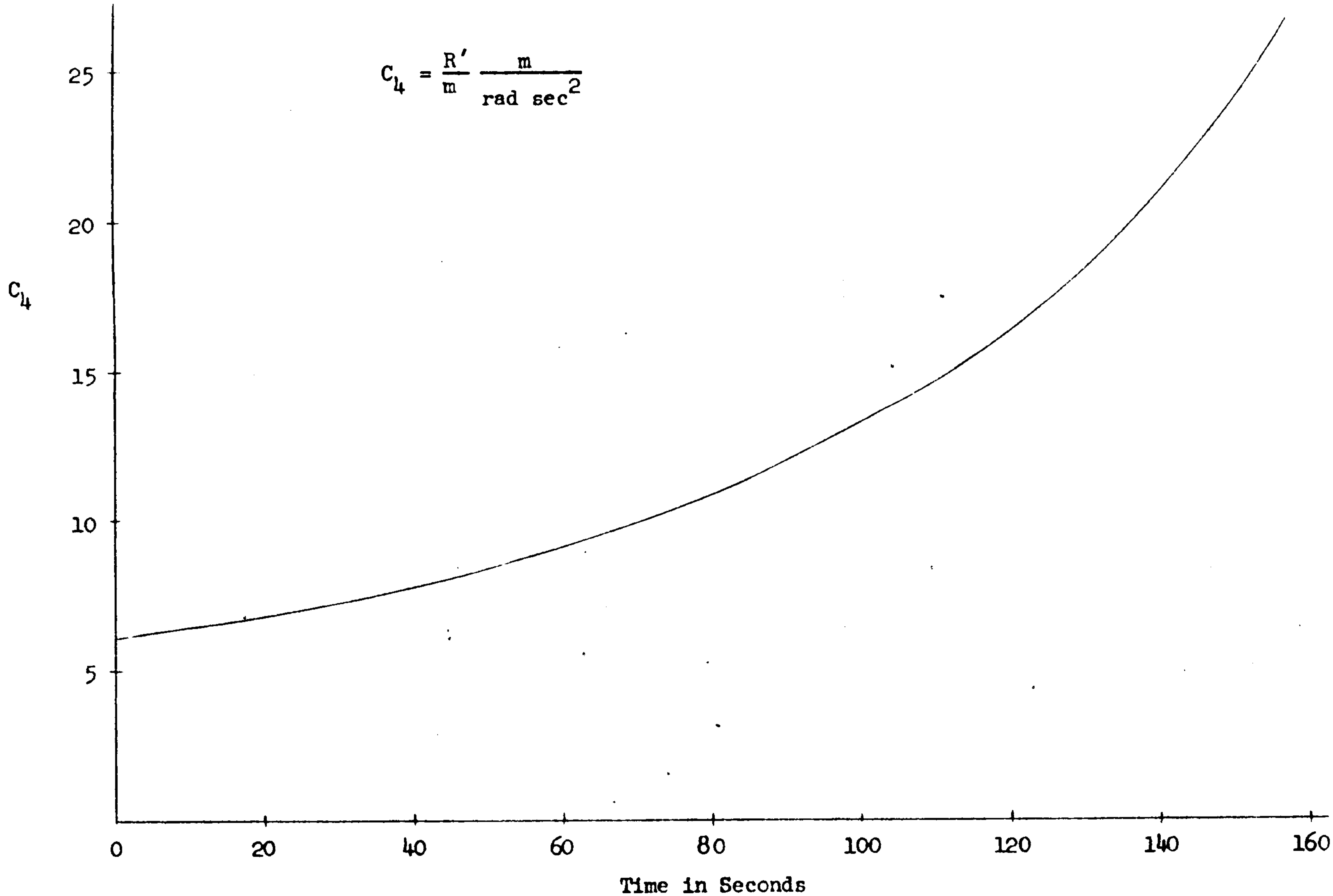


Figure B.6

Variation of C_4 with Time

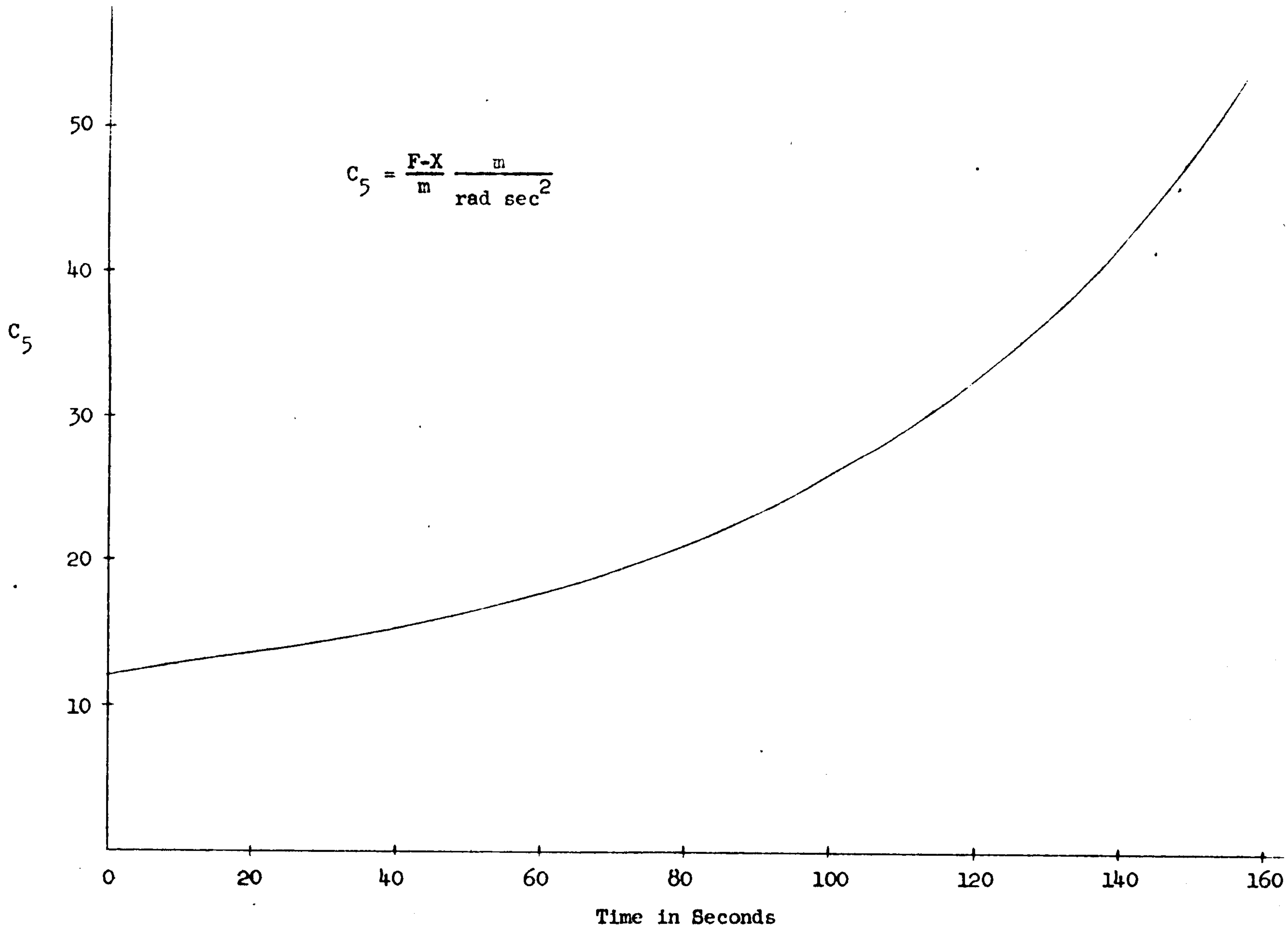


Figure B.7

Variation of C_5 with Time

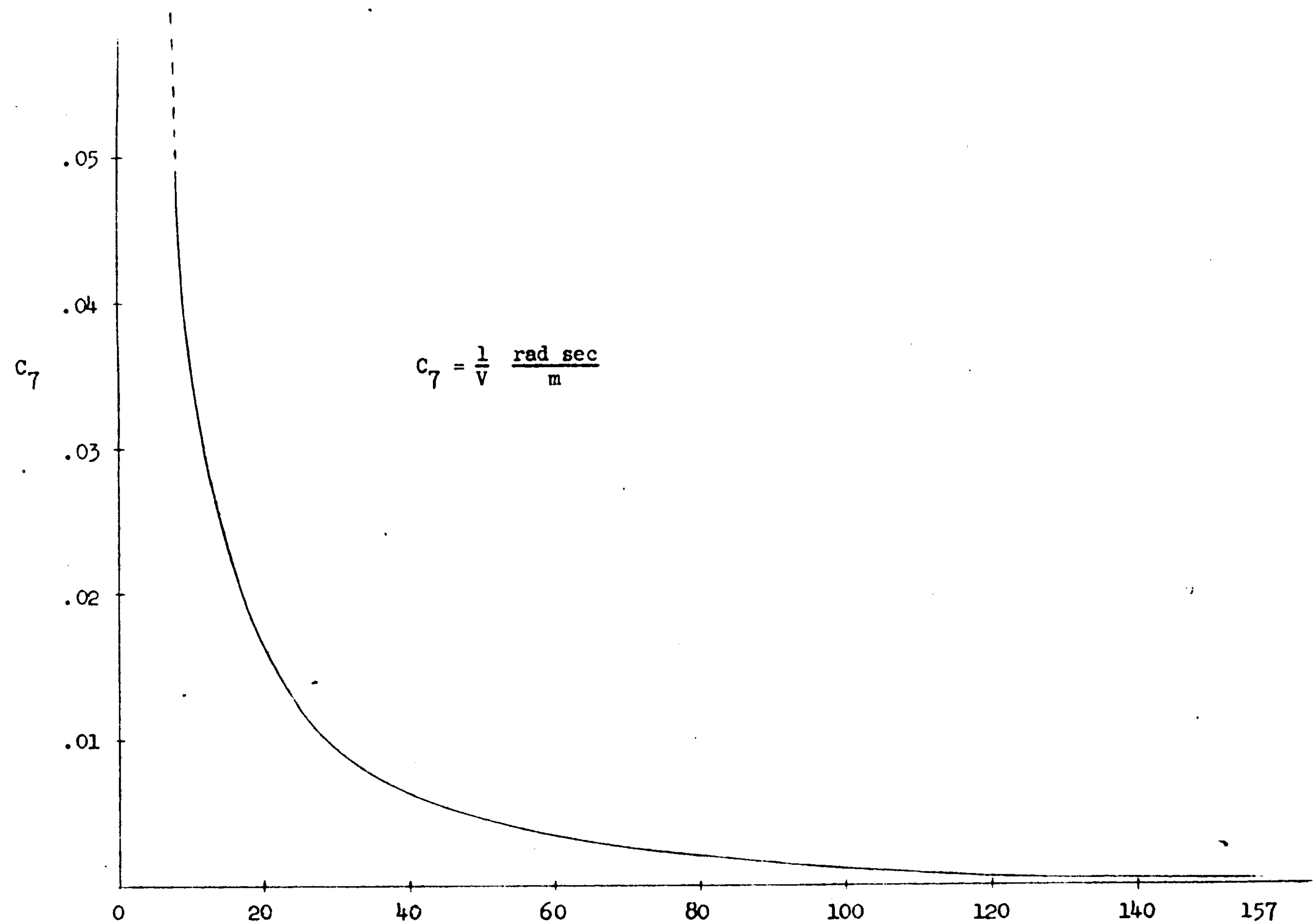


Figure B.8

Variation of C_7 with Time

$$\sin \upsilon = \frac{\dot{Z}}{V} = C_7\dot{Z} \cong \upsilon$$
$$\sin \alpha_w \cong \alpha_w \quad \text{(B.14)}$$

so

$$\alpha = \phi_R - C_7\dot{Z} + \alpha_w \quad \text{(B.15)}$$

The wind angle relationships for pitch and yaw planes are shown in Figure B.9. For small angles

$$\alpha_w = \frac{V_w \cos X}{V - V_w \sin X} \qquad \text{(pitch)}$$
$$\alpha_w = \frac{V_w}{V} \qquad \text{(yaw)} \quad \text{(B.16)}$$

For the Flexible Body, consider that the bending signals enter as corrections to the rigid body. The aerodynamic forces of drag and lift (X and N), are assumed to act just as they do on the rigid body, i.e., aerodynamic coupling is neglected. Figure B.10 shows the flexible body coordinate system, where only the first bending mode is considered. For the yaw plane we can write directly that

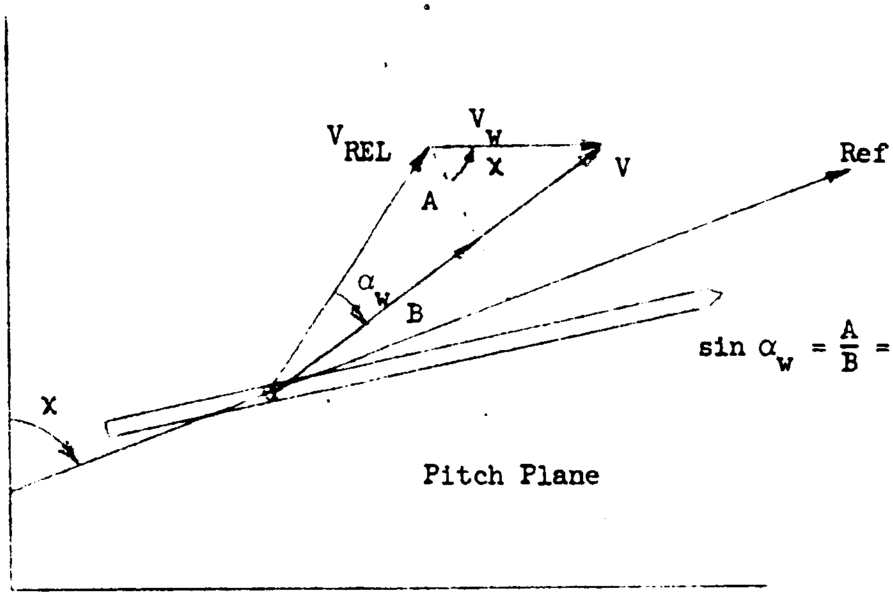
$$I_{xx}\ddot{\phi}_R = (X_{CP} - X_{CG})N + \frac{F}{2}(\ell_1 - \ell_2)$$
$$m\ddot{Z} = N \cos \phi_R - X \sin \phi_R + \frac{F}{2}(\sin \Theta_1 - \sin \Theta_2) \quad \text{(B.17)}$$

By considering the geometry of Fig. B.10 it can be found that for small angle assumptions on $Y_1'(X_\beta)\eta_1$, $\Theta_3 = Y_1'(X_\beta) - \beta_R$, and ϕ_R.

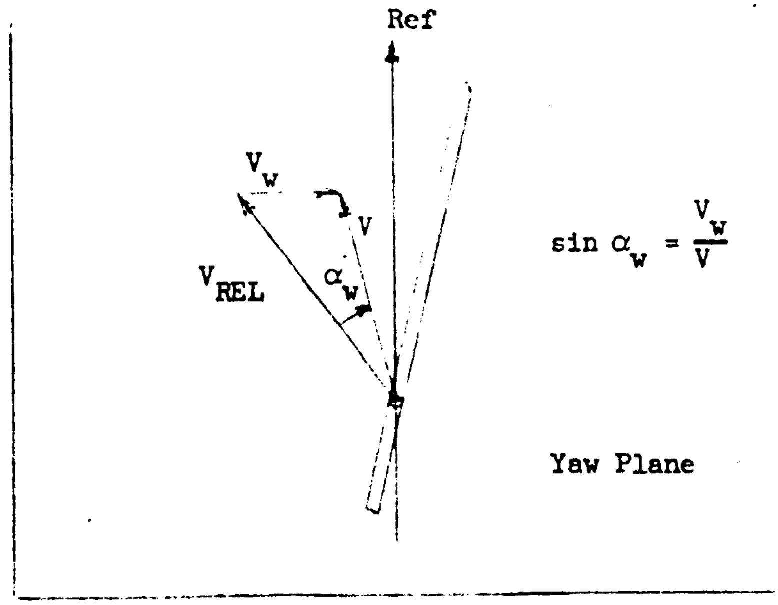
$$\ell_1 = Y_1'(X_\beta)\eta_1 (X_{CG} - X_\beta) - Y_1(X_\beta)\eta_1$$
$$\ell_2 = Y_1(X_\beta)\eta_1 - [Y_1'(X_\beta)\eta_1 - \beta_R] (X_{CG} - X_\beta)$$
$$\Theta_1 = \phi_R + \beta_R - Y_1'(X_\beta)\eta_1 \quad \text{(B.18)}$$
$$\Theta_2 = Y_1'(X_\beta)\eta_1 - \phi_R$$

and assuming small Θ_1 and Θ_2

$$\sin \Theta_1 - \sin \Theta_2 \cong \Theta_1 - \Theta_2 \quad \text{(B.19)}$$



$$\sin \alpha_W = \frac{A}{B} = \frac{V_W \cos X}{V - V_W \sin X}$$



$$\sin \alpha_W = \frac{V_W}{V}$$

Figure B.9

Wind Inputs for Pitch and Yaw Planes

Substituting equations (B.18) and (B.19) into (B.17) the <u>Flexible Body Equations</u> are (where additional flexible modes are added in)

$$\ddot{\phi}_R = - C_1\alpha - C_2\beta_R + \frac{F}{I_{xx}} \sum_i [Y_i'(X_\beta)\eta_i \; (X_{CG} - X_\beta) - Y_i(X_\beta)\eta_i]$$

(B.20)

$$\ddot{Z} = C_3\alpha + C_4\beta_R + C_5\phi_R - \frac{F}{m} \sum_i Y_i'(X_\beta)\eta_i$$

Sensors mounted on the vehicle are sensitive to the curvature due to bending, as well as the rigid body signals. That is

$$\phi(x) = \phi_R - \sum_i Y_i'(x)\eta_i$$

(B.21)

$$\dot{\phi}(x) = \dot{\phi}_R - \sum_i Y_i'(x)\dot{\eta}_i$$

and the angle of attack, as measured by its sensor is α_T where

$$\alpha_T = \alpha - \sum_i Y_i'(X_\alpha)\eta_i - C_7 \left[\sum_i Y_i(X_\alpha)\dot{\eta}_i + (X_\alpha - X_{CG}) \; \dot{\phi}_R \right] \qquad \text{(B.22)}$$

The bending equations are derived by assuming the vehicle is a free-free beam. The forced beam equation is

$$P(t, x) = m(x) \frac{d^2y}{dt^2} + \frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 y}{\partial x^2} \right] \qquad \text{(B.23)}$$

where $P(t, x)$ is the normal force and $y(t, x)$ is the normal displacement of station X. A solution is assumed to be of the form

$$y(t, x) = \sum_{i=1}^{\infty} Y_i(x) \; \eta_i(t) \qquad \text{(B.24)}$$

Substituting equation (B.24) into (B.23), multiplying by $Y_j(x)$ and integrating over the length yields

$$\int_0^L P(t, x)\, Y_j(x)\, dx = \sum_{i=1}^{\infty} \{\ddot{\eta}_i(t) \int_0^L m(x)\, Y_i(x)\, Y_j(x)\, dx +$$

$$+ \eta_i(t) \int_0^L [EI(x)\, Y_i''(x)]''\, Y_j(x)\, dx\} \qquad (B.25)$$

From examination of the homogeneous beam equation and the boundary conditions of zero shear and bending moment at each end of the free-free beam the following conditions arise: First

$$\int_0^L [EI(x)\, Y_i''(x)]''\, Y_j(x)\, dx = \omega_i^2 \int_0^L m(x)\, Y_i(x)\, Y_j(x)\, dx \qquad (B.26)$$

and then orthogonality of the $Y_i(x)$ with respect to the mass distribution

$$\int_0^L m(x)\, Y_i(x)\, Y_j(x)\, dx = 0 \qquad \text{for } i \neq j \qquad (B.27)$$

Defining

$$Q_i = \int_0^L P(t, x)\, Y_i(x)\, dx = \text{generalized force on the } i^{th} \text{ mode}$$

(B.28)

$$M_i = \int_0^L m(x)[Y_i(x)]^2\, dx = \text{generalized mass of the } i^{th} \text{ mode}$$

the Bending Equation for the i^{th} mode is

$$\ddot{\eta}_i + \omega_i^2\, \eta_i = \frac{Q_i}{M_i} \qquad (B.29a)$$

A damping term is inserted, with the ξ_i value being chosen empirically so equation (B.29a) becomes

$$\ddot{\eta}_i + 2\xi_i\, \omega_i\, \dot{\eta}_i + \omega_i^2\, \eta_i = \frac{Q_i}{M_i} \qquad (B.29b)$$

The generalized force on the i^{th} mode due to rigid and bending body causes are found from integration of

$$P(t,\ x) = R'\beta_R\delta(X_\beta) + qA\alpha \frac{\partial C_{Z\alpha}}{\partial x}(x) \tag{B.30}$$

with the result

$$Q_i = R'\beta_R Y_i(X_\beta) + \int_0^L qA\alpha \frac{\partial C_{Z\alpha}}{\partial x}(x)\ Y_i(x)\ dx \tag{B.31}$$

The bending moment at X for the rigid body is derived by considering the distributed aerodynamic and reactive forces along the vehicle, where

$$f_A(t,\ x) = qA\alpha \frac{\partial C_{Z\alpha}}{\partial x}$$

$$\tag{B.32}$$

$$f_R(t,\ x) = m(x)\ [\ddot{z} + (X - X_{CG})\ \ddot{\phi}_R]\quad x > X_\beta$$

The bending moment is found by integrating the moments over the vehicle,

$$BM(t,\ x) = \int_x^L (\xi - x)\ [f_A(t,\ \xi) - f_R(t,\ \xi)]\ d\xi \tag{B.33}$$

Define

$$I_1(t,\ x) = \int_x^L (\xi - x)\ q(t)\ A \frac{\partial C_{Z\alpha}}{\partial x}(\xi)\ d\xi$$

$$I_2(t,\ x) = \int_x^L (\xi - x)\ m(\xi)\ d\xi \tag{B.34}$$

$$I_3(t,\ x) = \int_x^L (\xi - x)\ (\xi - x_{CG})\ m(\xi)\ d\xi$$

Then

$$BM(t,\ x) = I_1(t,\ x)\alpha - I_2(t,\ x)\ddot{z} - I_3(t,\ x)\ \ddot{\phi}_R \tag{B.35}$$

Substituting for $\ddot{\phi}_R$ from equations (B.11) and for $\ddot{z}$

$$\ddot{z} = \ddot{Z} - \frac{F-X}{m}\ \phi_R = C_3\alpha + C_4\beta_R \tag{B.36}$$

The Bending Moment at time t and station X can be expressed as

$$BM(t, x) = K_1(t, x)\alpha + K_2(t, x)\,\beta_R \tag{B.37}$$

where

$$K_1(t, x) = I_1(t, x) + C_1(t)\,I_3(t, x) - C_3(t)\,I_2(t, x)$$

$$K_2(t, x) = C_2(t)\,I_3(t, x) - C_4(t)\,I_2(t, x) \tag{B.38}$$

At $t \cong 80$ seconds into the flight and at $x = 90$ meters the coefficients K_1 and K_2 are calculated to yield

$$BM = (1.6\alpha + 12.4\beta_R)\,10^6 \text{ Kg.m} \tag{B.39}$$

The above results furnish a set of vehicle equations to describe the vehicle in yaw or pitch plane considering it as a rigid body with bending signals superposed on it.

REFERENCES

1. R. L. Swaim, "A General Theory and Analysis of the Dynamic Stability of Flexible Bodied Missiles," ASD-TDR-62-627, Oct.,1962.

2. G. W. Johnson and F. G. Kilmer, "MARP Report on Ballistic Missile Flight Control Theory," Sections I-V, IBM Report No. 60-504-48, Jan. 26, 1961.

3. R. L. Bisplinghoff, H. Ashley, R. L. Halfman, Aeroelasticity, Addison-Wesley Publishing Co., Inc., Cambridge, Mass., 1958.

4. H. L. Runyan, "Overall Missile Dynamics," Lecture Notes of a course sponsored by the George Washington University in cooperation with Inst. of Environm. Sci. and Cent. for Prev. of Deterioration, Washington, D. C., Oct. 30, 1962.

5. "Model Vehicle No. 2 for Advanced Control Studies," Working Paper furnished by M.S.F.C., 1964.

APPENDIX C

ALGORITHMS AND SUBROUTINES FOR TWO-INTERVAL CONTROL UNDER THE MODIFIED OPTIMAL CONTROL POLICY VIA LINEAR PROGRAMMING

C.1 Introduction

In this appendix, as the title implies, algorithms and subroutines for two-interval control under the modified optimum control policy via linear programming are presented. Here it is assumed that the reader has familiarized himself with the material given in sections 1 through 6 of Chapter 5. In the end of the appendix a Fortran program for input and output and the corresponding data for solution of example 5.3 are included. This shall help in using the subroutines.

C.2 Algorithm I

The algorithm for solution of problem VI as stated in section 5.5 is as follows.

Given the plant equations as in Equations (5.9) and (5.10) and $\underline{x}(K) = 0$, $K = 0$,

(1) Construct the coefficient matrix as in Equation (5.28).

(2) Construct the right hand side vector as in Equation (5.28).

(3) Using Simplex methods solve for the unknowns such that the right hand side of Equation (5.29) is minimum.

(4) Store $\underline{x}(K + 1)$, and $u(K)$.

(5) Replace $\underline{x}(K)$ by $\underline{x}(K + 1)$.

(6) $K = K + 1$.

(7) If $\Delta.K < T$, go to 1; if $\Delta.K \geq T$, stop.

(Δ = sampling period, T = total period of interest.)

If the mathematical model of the system is described by a set of differential equations, it must be quantized using the procedure described in section 5.6.

In view of the above algorithm the following individual subroutines are developed. The use of the subroutines for solution of problem VI with M = 1, will be explained through algorithm II.

C.3 Subroutine QUANT

Identification

QUANT - Fortran (II) quantization subroutine.

Purpose

To quantize in time linear continuous system of the type

$\underline{\dot{x}} = A\underline{x} + \underline{b}u + \underline{c}$

Restriction

Maximum allowable order of the system is 10. For systems of higher order change the dimension statement.

Usage

The routine is entered by the following statement

CALL (NC, DELT, CTOL, AC, PHI, BC, CC, H, D, ITPHI)

where

Quantity	Input/Output	Dimension	Purpose
NC	IN	scalar	Order of the system.
DELT	IN	scalar	Quantization of the system.
CTOL	IN	scalar	Tolerance for the elements of PHI.
AC	IN	(NCxNC)	Coefficient matrix of the continuous system.
PHI	OUT	(NCxNC)	Coefficient matrix of the quantized system.
BC	IN	(NC)	Control variable coefficient vector.

Quantity	Input/Output	Dimension	Purpose
CC	IN	(NC)	Disturbance vector
H	OUT	(NC)	Corresponds to BC in quantized version.
D	OUT	(NC)	Corresponds to CC in quantized version.
ITPHI	OUT	scalar	Number of terms considered in the series for PHI.

Method

The method used for quantization is given in section 6 of Chapter 5.

Listing

Listing of the subroutine is given on the next page.

C.4 Subroutine LP2STD

Identification

LP2STD - Fortran (II) Subroutine.

Purpose

To formulate the two-interval control problem in standard form of linear programming. The routine essentially generated A1 and B1 (defined below).

Restrictions

A1 and B1 are generated by the subroutine as shown in Equations (C.1) and (C.2). A1 and B1 are obtained by adjoining Equation (5.29) to Equation (5.28) with zero as the first element of B1. Note that row 1 in A1 corresponds to the coefficients of the variables in the performance index. Rows 2 to 2n + 1 (inclusive) correspond to the coefficients of the plant equations. The remaining rows correspond to the inequalities, the

```
C                          SUBROUTINE QUANT

C     DISCRETIZE CONTINUOUS SYSTEM
      SUBROUTINE QUANT(NC,DELT,CTOL,AC,PHI,BC,CC,H,D,ITPHI)
      DIMENSION AC(10,10),PHI(10,10),BC(10),CC(10),H(10),D(10),
     1AC1(10,10),AC2(10,10),D1(10,10)
C     NC = ORDER OF COEFF. MATRIX
C     DELT QUANTIZATION PERIOD
C     CTOL = TOLERANCE FOR ELEMENTS OF PHI
C     AC = COEFFICIENT MATRIX OF CONTINUOUS SYSTEM
C     PHI= COEFF. MATRIX OF QUANTIZED SYSTEM
C     BC = CONTROL VARIABLE COEFFICIENT VECTOR
C     CC = DISTURBANCE VECTOR
C     H = CORRESPONDS TO BC IN QUANTIZED VERSION
C     D = CORRESPONDS TO CC IN QUANTIZED VERSION
C     ITPHI = NO. OF TERMS CONSIDERED IN THE SERIES FOR PHI
      DO 509 I=1,NC
      DO 509 J=1,NC
      IF(I-J)510,511,510
  510 AC1(I,J)=0.
      GO TO 512
  511 AC1(I,J)=1.0
  512 AC2(I,J)=DELT*AC(I,J)
      D1(I,J)=DELT*AC1(I,J)+(DELT/2.)*AC2(I,J)
  509 PHI(I,J)=AC1(I,J)+AC2(I,J)
      FACT=1.
  513 FACT=FACT+1.
      L=0
      DO 514 I=1,NC
      DO 514 J=1,NC
  514 AC1(I,J)=AC2(I,J)
      DO 515 I=1,NC
      DO 515 J=1,NC
      AC2(I,J)=0.
      DO 516 K=1,NC
  516 AC2(I,J)=AC2(I,J)+AC(I,K)*AC1(K,J)
      AC2(I,J)=(DELT/FACT)*AC2(I,J)
  515 CONTINUE
      DO 517 I=1,NC
      DO 517 J=1,NC
      D1(I,J)=D1(I,J)+(DELT/(FACT+1.))*AC2(I,J)
  517 PHI(I,J)=PHI(I,J)+AC2(I,J)
      IF(FACT- 5.)513,523,523
  523 DO 518 I=1,NC
      DO 518 J=1,NC
      IF (ABSF(AC2(I,J))-CTOL)519,518,518
  519 L=L+1
  518 CONTINUE
      IF(L-NC*NC)520,521,521
  520 GO TO 513
  521 DO 522 I=1,NC
      H(I)=0.0
      D(I)=0.0
      DO 522 J=1,NC
      D(I)=D(I)+D1(I,J)*CC(J)
  522 H(I)=D1(I,J)*BC(J)+H(I)
      ITPHI=FACT
      RETURN
      END
```

first four inequalities being the constraints on u(K) and u(K+1) - the control signals. This order must be preserved. Non-zero elements in the rectangular blocks in Equations (C.1) and (C.2) must be read in externally before calling LP2STD. The dimensions of the various variables involved are restricted by the dimension statement (see the listing of the subroutine).

$$A1 = \begin{bmatrix} \tilde{\underline{c}}'(K+1) & \tilde{\underline{c}}'(K+2) & \tilde{\underline{\gamma}}'(K) & \tilde{\underline{\gamma}}'(K+1) & 0 & 0 \\ \tilde{I} & 0 & -\tilde{H}(K) & 0 & 0 & 0 \\ -\tilde{A}(K+1) & \tilde{I} & 0 & -\tilde{H}(K+1) & 0 & 0 \\ \tilde{B}_{1n}(K+1) & 0 & \tilde{B}_{n,n+1}(K+1) & 0 & I & 0 \\ 0 & \tilde{B}_{1n}(K+2) & 0 & \tilde{B}_{n,n+1}(K+2) & 0 & I \end{bmatrix} \quad (C.1)$$

$$B1 = \begin{bmatrix} 0 \\ \underline{P}(K) \\ \underline{d}(K+1) \\ \underline{\alpha}(K+1) \\ \underline{\alpha}(k+2) \end{bmatrix} \quad (C.2)$$

<u>Usage</u>

The routine is entered by the following statement:

CALL LP2STD (N, NN, M, XIC, H1, H2, D1, D2, PHI1, PHI2, A, B, A1, B1)

where

Quantity	Input/Output	Dimension	Purpose
N	IN	scalar	order of the system
NN	IN	scalar	Number of the columns of A-matrix
M	IN	scalar	Number of the rows of A-matrix

Quantity	Input/Output	Dimension	Purpose
XIC	IN	(N)	Initial conditions for the dynamic system.
H1	IN	(N)	Control coefficient vector for t = t(K).
H2	IN	(N)	Control coefficient vector for t = t(K+1).
D1	IN	(N)	Disturbance vector for t = t(K).
D2	IN	(N)	Disturbance vector for t = t(K+1).
PHI 1	IN	(NxN)	Coefficient matrix for t = t(K).
PHI 2	IN	(NxN)	Coefficient matrix for t = t(K+1).
A	OUT	(MxNN)	Coefficient matrix of L.P. problem in standard form.
B	OUT	(M)	Right hand side vector for L.P. problem in standard form.
A1	IN/OUT	(MxNN)	Same as A.
B1	IN/OUT	(MxNN)	Same as B.

For time-invariant systems H1 = H2, D1 = D2 and PHI 1 = PHI 2.

Listing

The listing of the subroutine is given on the next page.

```
C                        SUROUTINE LP2STD

C     2-INTERVAL L.P. PROBLEM IN STANDARD FORM
      SUBROUTINE LP2STD(N,NN,M,XIC,H1,H2,D1,D2,PHI1,PHI2,A,B,A1,B1)
C     N= ORDER OF THE SYSTEM
C     NN= NO. OF COLUMNS OF A-MATRIX
C     M= NO. OF ROWS OF A-MATRIX
C     XIC=INITIAL CONDITIONS FOR THE DYNAMIC SYSTEM
C     H1= CONTROL COEFF. VECTOR FOR T=T(K)
C     H2= CONTROL COEFF. VECTOR FOR T=T(K+1)
C     D1= DISTURBANCE VECTOR FOR T=T(K)
C     D2= DISTURBANCE VECTOR FOR T=T(K+1)
C     PHI1= COEFF. MATRIX FOR T=T(K)
C     PHI2= COEFF. MATRIX FOR T=T(K+1)
C     A= COEFF. MATRIX OF L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C     B= RHS VECTOR FOR L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C     A1= SAME AS A (INPUT TO SUBROUTINE)
C     B1= SAME AS B (INPUT TO SUBROUTINE)
      DIMENSION XIC(10),H1(10),H2(10),D1(10),D2(10),B(40),B1(40),
     1PHI1(10,10),PHI2(10,10),A1(40,60),A(40,60),B2(11)
C     H2= H- VECTOR FOR T=T(K+1)
C     D1= DISTURBANCE VECTOR FOR T=T(K)
C     D2= DISTURBANCE VECTOR FOR T=T(K+1)
C     A= COEFF. MATRIX OF L.P. PROBLEM IN STANDARD FORM
C     B= RHS VECTOR FOR L.P. PROBLEM IN STANDARD FORM
C     N= ORDER OF THE DYNAMIC SYSTEM
C     NN= NO. OF COLUMNS OF A-MATRIX
C     M= NO. OF ROWS OF A-MATRIX
C     XIC=INITIAL CONDITIONS FOR THE DYNAMIC SYSTEM
C
C     CONSTRUCTION OF A-MATRIX STARTS
C     ALL +1 OR -1 DUE TO STATE VARIABLES
C
      I5=2*N+1
      DO 12 I=2,I5
      J2=2*(I-1)
      J1=J2-1
      A1(I,J1)=+1.0
      A1(I,J2)=-1.0
   12 CONTINUE
C
C     COEFFICIENTS WHICH ARE ELEMENTS OF STATE TRANSITION MATRIX
C
      I6=N+2
      DO 13 I=I6,I5
      I1=I-(N+1)
      DO 13 J=1,N
      J1=2*J-1
      J2=2*J
      A1(I,J1)=-PHI2(I1,J)
      A1(I,J2)=+PHI2(I1,J)
   13 CONTINUE
C
C     COEFFICIENTS WHICH ARE ELEMENTS OF H- VECTOR
C
      I1=N+1
      DO 14 I=2,I1
      J1=4*N+1
      J2=4*N+2
```

```
      A1(I,J1)=-H1(I-1)
      A1(I,J2)=+H1(I-1)
   14 CONTINUE
      I1=N+2
      I2=2*N+1
      J1=4*N+3
      J2=4*N+4
      DO 15 I=I1,I2
      I3=I-N-1
      A1(I,J1)=-H2(I3)
      A1(I,J2)=+H2(I3)
   15 CONTINUE
C
C     COEFFICIENTS DUE TO CONSTRAINTS ON CONTROL VARIABLE
C
      I2=2*N+2
      I3=I2+1
      I4=I2+2
      I5=I2+3
      J1=4*N+1
      J2=J1+1
      J3=J1+2
      J4=J1+3
      A1(I2,J1)=+1.0
      A1(I2,J2)=-1.0
      A1(I3,J1)=-1.0
      A1(I3,J2)=+1.0
      A1(I4,J3)=+1.0
      A1(I4,J4)=-1.0
      A1(I5,J3)=-1.0
      A1(I5,J4)=+1.0
C
C     SLACK VARIABLES COEFFICIENTS DUE TO ALL CONSTRAINTS
C
      I1=2*N+2
      J1=4*N+4
      DO 16 I=I1,M
      J=2*N+3+I
      A1(I,J)=+1.0
   16 CONTINUE
C     A1 .. COEFFICIENTS DUE TO CONSTRAINTS OTHER THAN THE CONTROL
C     VARIABLE MUST BE READ EXTERNALLY
C
C     CONSTRUCTION OF B1
C
      I1=N+1
      DO 25 I=2,I1
   25 B2(I)=0.0
      DO 17 I=2,I1
      I2=I-1
      DO 18 J=1,N
   18 B2(I)=B2(I)+PHI1(I2,J)*XIC(J)
   17 B1(I)=B2(I)+D1(I2)
      I1=N+2
      I2=N*2+1
      DO 19 I=I1,I2
      J=I-N-1
   19 B1(I)=D2(J)
C
C     B1 .. ELEMENTS DUE TO ALL CONSTRAINTS MUST BE READ EXTERNALLY
C
```

KOUT (3) = Number of iterations since last inversion (ignoring final inversion if done).

KOUT (4) = Number of inversions done (including final and initial inversions).

KOUT (5) = Number of pivots done.

KOUT (6) = Infeasibility flag, 1 = infeasible; 0 = feasible.

KOUT (7) = Final pivot column selected.

The NXZ components of XZ are as follows:

XZ (1) through XZ (N) = State variables corresponding to t = t(K+1).

XZ (N+1) through XZ (2N) = State variables corresponding to t = t(K+2).

XZ (2N+1) = Control variable for t = t(K).

XZ (2N+2) = Control variable for t = t(K+1).

XZ (2N+3) through XZ (NXZ) = "r" slack variables.

<u>Listing</u>

The listing of the subroutine LP2MAS is given on the next page.

```
C                          SUBROUTINE LP2MAS
C     MAIN SUBROUTINE FOR TWO INTERVAL CONTROL VIA LINEAR PROGRAMMING
      SUBROUTINE LP2MAS(INFIX,TOL,HEDER,AC,BC,CC,PHI,H,D,XZI,A,B,A1,B1,
     1KOUT,Z,XZ,NXZ)
C     INFIX=INPUT (VECTOR) TO SIMPLX SUBROUTINE
C     TOL=  INPUT (VECTOR) TO SIMPLX SUBROUTINE
C     HEDER=INPUT (VECTOR)
C     HEDER(1)=QUANTIZATION PERIOD
C     HEDER(2)= TOLERANCE FOR QUANTIZATION
C     HEDER(3)= ORDER OF DYNAMIC SYSTEM
C     AC= COEFF. MATRIX FOR CONTINUOUS SYSTEM
C     BC = CONTROL VARIABLE COEFFICIENT VECTOR
C     CC = DISTURBANCE VECTOR
C     PHI= COEFF. MATRIX FOR QUANTIZED SYSTEM
C     H= CONTROL COEFF. VECTOR (QUANTIZED)
C     D= DISTURBANCE VECTOR (QUANTIZED)
C     XZI=INITIAL CONDITIONS FOR THE DYNAMIC SYSTEM
C     A= COEFF. MATRIX OF L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C     B= RHS VECTOR FOR L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C     A1= SAME AS A (INPUT TO SUBROUTINE)
C     B1= SAME AS B (INPUT TO SUBROUTINE)
C     KOUT= OUTPUT CONDITIONS FOR SIMPLEX SOLUTION
C     Z= SOLUTION VECTOR OF NON-NEGATIVE VARIABLES
C     XZ= SOLUTION VECTOR
C     NXZ= NUMBER OF VARIABLES IN THE SOLUTION VECTOR XZ
      DIMENSION INFIX(8),TOL(4),AC(10,10),BC(10),CC(10),PHI(10,10),
     1H(10),D(10),XZI(10),A(40,60),B(40),A1(40,60),B1(40),KOUT(7),
     2Z(60),XZ(60),HEDER(5),ERS(8),JH(40),X(40),P(40),Y(40),KB(60),
     3E(40,40)
      N=INFIX(2)
      M=INFIX(4)
      TQUAN=HEDER(1)
      TOLER=HEDER(2)
      N1=HEDER(3)
      PRM=0.0
      CALL QUANT(N1,TQUAN,TOLER,AC,PHI,BC,CC,H,D,ITR)
      CALL LP2STD(N1,N,M,XZI,H,H,D,D,PHI,PHI,A,B,A1,B1)
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERS,JH,X,P,Y,KB,E)
      DO 10 I=1,N
   10 Z(I)=0.0
      DO 11 I=2,M
      JHH=JH(I)
      IF(JHH)11,11,12
   12 Z(JHH)=X(I)
   11 CONTINUE
      N2=2*N1+2
      DO 13 I=1,N2
   13 XZ(I)=Z(2*I-1)-Z(2*I)
      I1=2*N1+3
      NXZ=N-2*N1-2
      DO 14 I=I1,NXZ
      J=I1+I-1
   14 XZ(I)=Z(J)
C     VALUES OF THE STATE VARIABLES AND THE SLACK VARIABLES HAVE
C     BEEN STORED IN VECTOR XZ
      RETURN
      END
```

C.7 Subroutine LP2MAT

This subroutine is identical to LP2MAS in all respects, except it is for time-varying systems. The calling sequence for this subroutine is

```
CALL LP2MAT ( INFIX, TOL, HEDER, AC, BC, CC, PHI 1, H1, D1, XZ1,
     A, B, A1, B1, KOUT, Z, XZ, NXZ ).
```

The listing of the subroutine is given on the next page.

```
C                              SUBROUTINE LP2MAT

C      MAIN SUBROUTINE FOR TWO INTERVAL CONTROL BY LINEAR PROGRAMMING
C      FOR TIME VARYING SYSTEM
C      INFIX=INPUT (VECTOR) TO SIMPLX SUBROUTINE
C      TOL=  INPUT (VECTOR) TO SIMPLX SUBROUTINE
C      HEDER=INPUT (VECTOR)
C      HEDER(1)=QUANTIZATION PERIOD
C      HEDER(2)= TOLERANCE FOR QUANTIZATION
C      HEDER(3)= ORDER OF DYNAMIC SYSTEM
C      AC= COEFF. MATRIX FOR CONTINUOUS SYSTEM
C      BC = CONTROL VARIABLE COEFFICIENT VECTOR
C      CC = DISTURBANCE VECTOR
C      PHI1= COEFF. MATRIX FOR T=T(K)
C      H1= CONTROL COEFF. VECTOR FOR T=T(K)
C      D1= DISTURBANCE VECTOR FOR T=T(K)
C      XZI=INITIAL CONDITIONS FOR THE DYNAMIC SYSTEM
C      A= COEFF. MATRIX OF L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C      B= RHS VECTOR FOR L.P. PROBLEM IN STANDARD FORM (OUTPUT OF SUB.)
C      A1= SAME AS A (INPUT TO SUBROUTINE)
C      B1= SAME AS B (INPUT TO SUBROUTINE)
C      KOUT= OUTPUT CONDITIONS FOR SIMPLEX SOLUTION
C      Z= SOLUTION VECTOR OF NON-NEGATIVE VARIABLES
C      XZ= SOLUTION VECTOR
C      NXZ= NUMBER OF VARIABLES IN THE SOLUTION VECTOR XZ
       SUBROUTINE LP2MAT(INFIX,TOL,HEDER,AC,BC,CC,PHI1,H1,D1,XZI,A,B,A1,
      1B1,KOUT,Z,XZ,NXZ)
       DIMENSION INFIX(8),TOL(4),AC(10,10),BC(10),CC(10),PHI1(10,10),
      1H1(10),      XZI(10),A(40,60),B(40),A1(40,60),B1(40),KOUT(7),
      2Z(60),XZ(60),HEDER(5),ERS(8),JH(40),X(40),P(40),Y(40),KB(60),
      3E(40,40),D1(10),D2(10),H2(10),PHI2(10,10)
       N=INFIX(2)
       M=INFIX(4)
       TQUAN=HEDER(1)
       TOLER=HEDER(2)
       N1=HEDER(3)
       PRM=0.0
       CALL QUANT(N1,TQUAN,TOLER,AC,PHI2,BC,CC,H2,D2,ITR)
       CALL LP2STD(N1,N,M,XZI,H1,H2,D1,D2,PHI1,PHI2,A,B,A1,B1)
       CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERS,JH,X,P,Y,KB,E)
       DO 10 I=1,N
    10 Z(I)=0.0
       DO 11 I=2,M
       JHH=JH(I)
       IF(JHH)11,11,12
    12 Z(JHH)=X(I)
    11 CONTINUE
       N2=2*N1+2
       DO 13 I=1,N2
    13 XZ(I)=Z(2*I-1)-Z(2*I)
       I1=2*N1+3
       NXZ=N-2*N1-2
       DO 14 I=I1,NXZ
       J=I1+I-1
    14 XZ(I)=Z(J)
       DO 15 I=1,N1
       DO 16 J=1,N1
    16 PHI1(I,J)=PHI2(I,J)
```

```
      H1(I)=H2(I)
   15 D1(I)=D2(I)
C     VALUES OF THE STATE VARIABLES AND THE SLACK VARIABLES HAVE
C     BEEN STORED IN VECTOR XZ
      RETURN
      END
```

C.8 Algorithm II

Subroutine LP2MAS (or LP2MAT for time-varying systems) together with QUANT, LP2STD and SIMPLX incorporate the first four steps of algorithm I. So using these subroutines, the algorithm I can be rewritten as follows:

1. Read the input data necessary for LP2MAS (or LP2MAT) including XZI = X(K), K = 0.
2. Call LP2MAS (or LP2MAT).
3. Store $\underline{x}(K + 1)$, u(K).
4. Replace XZI by $\underline{x}(K + 1)$.
5. If $\Delta.K < T$, go to 6; if $\Delta.K \geq T$, stop.
6. Go to 2 for time-invariant system; go to 1 for time-varying system.

C.9 Example

Use the above algorithm to solve the optimum control problem as stated in example 3 of Chapter 5, for a period of 10 seconds with quantization period of 0.1 second.

The Fortran program for input-output statements and the calling of the subroutine LP2MAS, is listed below. Following this program, the data is listed. The solution obtained is shown in Figure 5.10.

```
C                      SOURCE PROGRAM FOR INPUT-OUTPUT
      DIMENSION INFIX(8),TOL(4),HEDER(6),AC(10,10),BC(10),CC(10),
     1PHI(10,10),H(10),D(10),XZI(10),XZO(10),A(40,60),B(40),A1(40,60)
     2B1(40),KOUT(7),Z(60),XZ(60),NUT(10)
      READ INPUT TAPE 5,200,(NUT(I),I=1,10)
      READ INPUT TAPE 5,201,(INFIX(I),I=1,8)
      READ INPUT TAPE 5,203,(HEDER(I),I=1,6)
      READ INPUT TAPE 5,202,(TOL(I),I=1,4)
      N=INFIX(2)
      M=INFIX(4)
      N1=HEDER(3)
      NON1=HEDER(4)
      NON2=HEDER(5)
      READ INPUT TAPE 5,203,(XZO(I),I=1,N1)
      READ INPUT TAPE 5,203,(BC(I),I=1,N1)
      READ INPUT TAPE 5,203,(CC(I),I=1,N1)
      READ INPUT TAPE 5,203,((AC(I,J),J=1,N1),I=1,N1)
      CC(2)=CC(2)*HEDER(6)
      CC(3)=CC(3)*HEDER(6)
      DO 19 I=1,M
      B1(I)=0.0
      DO 19 J=1,N
   19 A1(I,J)=0.0
      DO 20 I=1,NON2
   20 READ INPUT TAPE 5,204,I1,(B1(I1))
      B1(12)=B1(12)-HEDER(6)
      B1(13)=B1(13)+HEDER(6)
      B1(14)=B1(14)-HEDER(6)
      B1(15)=B1(15)+HEDER(6)
      B1(16)=B1(16)-1.845*HEDER(6)
      B1(17)=B1(17)+1.845*HEDER(6)
      B1(18)=B1(16)
      B1(19)=B1(17)
      DO 21 I=1,NON1
   21 READ INPUT TAPE 5,205,I1,J1,(A1(I1,J1))
  200 FORMAT(10I5)
  201 FORMAT(8I5)
  202 FORMAT(4E15.8)
  203 FORMAT(6F12.6)
  204 FORMAT(I3,F15.8)
  205 FORMAT(2I3,F15.8)
      WRITE OUTPUT TAPE 6,210,(I,NUT(I),I=1,10)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,211,(I,INFIX(I),I=1,8)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,212,(I,HEDER(I),I=1,6)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,213,(I,TOL(I),I=1,4)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,217,(I,XZO(I),I=1,N1)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,214,(I,BC(I),I=1,N1)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,215,(I,CC(I),I=1,N1)
      WRITE OUTPUT TAPE 6,100
      WRITE OUTPUT TAPE 6,216,((I,J,AC(I,J),J=1,N1),I=1,N1)
      WRITE OUTPUT TAPE 6,100
```

```
210 FORMAT(10(4HNUT(,I2,3H )=,I3,1X))
211 FORMAT(8(6HINFIX(,I2,3H )=,I4,1X))
212 FORMAT(3(6HHEDER(,I2,3H )=,F15.8,2X))
213 FORMAT(4(4HTOL(,I2,3H )=,E15.8,2X))
214 FORMAT (6(3HBC(,I2,3H )=,F12.6,2X))
215 FORMAT (6(3HCC(,I2,3H )=,F12.6,2X))
216 FORMAT(5(3HAC(,I2,1H,,I2,3H )=,F12.6,2X))
217 FORMAT (6(4HXZO(,I2,3H )=,F11.6,2X))
    ITRAT=0
    ZED=0.0
    DO 51 I=1,N1
 51 XZI(I)=XZO(I)
    IIT=0
    IT=1
    ITMAX=NUT(3)
    DO 150 I=1,ITMAX
    IF(IT-(NUT(4)+1))27,28,28
 28 IT=1
 27 CONTINUE
C
    CALL LP2MAS(INFIX,TOL,HEDER,AC,BC,CC,PHI,H,D,XZI,A,B,
   1A1,B1,KOUT,Z,XZ,NXZ)
    PI=5.0*(ABSF(XZ(1))+ABSF(XZ(4)))+0.02*(ABSF(XZ(3))+ABSF(XZ(6)))
    ITRAT=ITRAT+1
    IF(NUT(1)-ITRAT)54,55,55
 55 WRITE OUTPUT TAPE 6,221,((I,J,PHI(I,J),J=1,N1),I=1,N1)
    WRITE OUTPUT TAPE 6,100
    WRITE OUTPUT TAPE 6,222,(I,H(I),I=1,N1)
    WRITE OUTPUT TAPE 6,100
    WRITE OUTPUT TAPE 6,223,(I,D(I),I=1,N1)
    WRITE OUTPUT TAPE 6,100
 54 CONTINUE
    IF(NUT(2)-ITRAT)56,57,57
 57 WRITE OUTPUT TAPE 6,101
    WRITE OUTPUT TAPE 6,224,(I,Z(I),I=1,N)
    WRITE OUTPUT TAPE 6,100
    WRITE OUTPUT TAPE 6,225,((I,J,A(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 6,100
    WRITE OUTPUT TAPE 6,226,(I,B(I),I=1,M)
    WRITE OUTPUT TAPE 6,101
 56 CONTINUE
    IF(IT-1)25,26,25
 26 IIT=IIT+1
    TII=IIT
    DUMMY=NUT(4)
    TIME=((DUMMY*TII)-(DUMMY-1.))*HEDER(1)
    ALPH1=XZ(1)-0.00192*XZ(3)+HEDER(6)
    ALPH2=XZ(4)-0.00192*XZ(6)+HEDER(6)
    BM1=+1.845*XZ(1)-.00355*XZ(3)+12.380*XZ(7)+1.845*HEDER(6)
    BM2=+1.845*XZ(4)-.00355*XZ(6)+12.380*XZ(8)+1.845*HEDER(6)
    ZED=ZED+XZ(3)*HEDER(1)
    IF(NUT(5)-1)31,32,33
 31 CONTINUE
    WRITE OUTPUT TAPE 6,229,ITRAT
    WRITE OUTPUT TAPE 6,227,(I,KOUT(I),I=1,7)
    WRITE OUTPUT TAPE 6,100
    WRITE OUTPUT TAPE 6,228,(I,XZ(I),I=1,NXZ)
    WRITE OUTPUT TAPE 6,230,ALPH1,ALPH2,BM1,BM2,ZED
    WRITE OUTPUT TAPE 6,100
 33 NUT(5)=NUT(5)-1
    WRITE OUTPUT TAPE 6,234
```

```
   32 WRITE OUTPUT TAPE 6,235,TIME,XZ(1),XZ(2),XZ(3),XZ(7),
     1ALPH1,BM1,KOUT(1),ZED
   25 IT=IT+1
      DO 58 I=1,N1
   58 XZ1(I)=XZ(I)
      IF(3-KOUT(1))151,150,151
  221 FORMAT(4(4HPHI(,I2,1H,,I2,3H )=,E15.8,2X))
  222 FORMAT(5(2HH(,I2,3H )=,E15.8,2X))
  223 FORMAT(5(2HD(,I2,3H )=,E15.8,2X))
  224 FORMAT (5(2HZ(,I2,3H )=,E15.8,2X))
  225 FORMAT(6(2I3,F12.6,2X))
  226 FORMAT(7(I3,F12.6,2X))
  227 FORMAT(1H0,/7(5HKOUT(,I2,3H )=,I5,2X))
  228 FORMAT (5(3HXZ(,I2,3H )=,E15.8,2X))
  229 FORMAT(15HITERATION NO. =,I5//)
  100 FORMAT(1H0)
  101 FORMAT(1H1)
  150 CONTINUE
  151 CONTINUE
C
  230 FORMAT(9HALPHA(1)=,E15.8,2X,9HALPHA(2)=,E15.8,2X,6HBM(1)=,E15.8,
     1 2X,6HBM(2)=,E15.8,2X,4HZED=,E15.8)
  231 FORMAT(1H1)
  233 FORMAT(2F12.8)
  234 FORMAT(4X,4HTIME,12X,3HPHI,10X,7HDPHI/DT,9X,5HDZ/DT,10X,4HBETA,
     111X,5HALPHA,11X,4HB.M.,7X,7HKOUT(1),9X,2H Z///)
  235 FORMAT(3X,F7.2,5X,6(E13.6,2X),I5)
 1000 CONTINUE
C
      CALL EXIT
      END
```

D A T A

```
    1     1   101     1     2
    4    32    40    23     2     1   100     0
 0.1          0.00000001   3.0              64.0          16.          0.1744
+0.10000000E-08+0.10000000E-08-0.10000000E-06+0.10000000E-10
  0.          0.0          0.0
 0.0          -0.448        10.93
  0.0          0.0791       5.59
  0.          1.0          0.0              0.0791        0.0          -.000153
 26.59         0.0          -.0108
  8+0.0872
  9+0.0872
 10+0.0872
 11+0.0872
 12+0.262
 13+0.262
 14+0.262
 15+0.262
 16+2.24
 17+2.24
 18+2.24
 19+2.24
 20+.1744
 21+.1744
```

```
22+.1744
23+.1744
 1   1+5.0
 1   2+5.0
 1   3+0.0
 1   4+0.0
 1   5+0.02
 1   6+0.02
 1   7+5.0
 1   8+5.0
 1   9+0.0
 1  10+0.0
 1  11+0.02
 1  12+0.02
 1  13+0.0
 1  14+0.0
 1  15+0.0
 1  16+0.0
12   1+1.0
12   2-1.0
13   1-1.0
13   2+1.0
14   7+1.0
14   8-1.0
15   7-1.0
15   8+1.0
12   5-0.00192
12   6+0.00192
13   5+0.00192
13   6-0.00192
14  11-0.00192
14  12+0.00192
15  11+0.00192
15  12-0.00192
16   1+1.845
16   2-1.845
17   1-1.845
17   2+1.845
18   7+1.845
18   8-1.845
19   7-1.845
19   8+1.845
16   5-.00355
16   6+.00355
17   5+.00355
17   6-.00355
18  11-.00355
18  12+.00355
19  11+.00355
19  12-.00355
16  13+12.38
16  14-12.38
17  13-12.38
17  14+12.38
18  15+12.38
18  16-12.38
19  15-12.38
19  16+12.38
20   1+1.0
20   2-1.0
21   1-1.0
```

21	2+1.0
22	7+1.0
22	8-1.0
23	7-1.0
23	8+1.0

C.10 Recommendations to Improve Algorithm II

The truncation error encountered in the SIMPLX routine is cumulative. As a result, if the states are calculated substituting the control variable in the plant equations, they differ slightly from the results obtained using the above algorithm. The difference increases as the interval over which the solution is found increases. This error can be avoided by the following improvement in algorithm II.

C.11 Algorithm III

Steps 1 and 2 as in algorithm II.

3(a). Substitute u(K) in the plant equations and calculate $\underline{x}(K+1)$.

3(b). Store u(K) and $\underline{x}(K+1)$ as calculated in 3(a).

Steps 4, 5, and 6 are the same as before.

REFERENCE

1. HO RS MSUB Linear Programming Subroutine from SHARE Library.

APPENDIX D

A STEEP DESCENT PROCEDURE FOR MINIMIZATION PROBLEMS

A steepest descent procedure due to A. V. Balakrishnan [1] which guarantees convergence of the procedure to a solution of the minimization of a quadratic performance index subject to a set of constraining linear differential equations and satisfying the conditions of fixed initial time and position, fixed final time and free final end-point, was adapted to the formulation of the minimum problem with fixed wind for MV2 in the seventh and tenth monthly progress reports. This procedure has the advantage not only of guaranteed convergence, but also of convergence to the unique solution having the least control effort, i.e., to the solution for which

$$\int_{t_o}^{t_1} ||u||^2(t)\,dt \text{ is a minimum.}$$

For the sake of completeness we present this procedure here. It is desired to minimize the functional,

$$J = \int_{t_o}^{t_1} [\,||x(t)||^2 + k\ ||u(t)||^2]\,dt \tag{D.1}$$

where

$$||x(t)||^2 = \sum_{i=1}^{n} x_i^2(t)$$

$$||u(t)||^2 = \sum_{j=1}^{m} u_j^2(t)$$

and the x_i's and u_j's are related by the linear differential equations,

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}(t)\, x_j(t) + \sum_{\ell=1}^{m} b_{i\ell}(t)\, u_j(t), \qquad i = 1, 2, \ldots, n \tag{D.2}$$

satisfying the initial conditions,

$$x_i(t_o) = C_i \quad , \qquad i = 1, 2, \ldots, n \tag{D.3}$$

and subject to the constraints,

$$\int_{t_o}^{t_1} ||\, u(t)\, ||^2 \, dt \leq M^2 \tag{D.4}$$

We shall employ the following notation. $A(t)$ will denote the square matrix

$$\begin{bmatrix} a_{11}(t) & \ldots & a_{1n}(t) \\ \ldots & \ldots & \ldots \\ a_{n1}(t) & \ldots & a_{nn}(t) \end{bmatrix} ,$$

$B(t)$ will denote the rectangular matrix

$$\begin{bmatrix} b_{11}(t) & \ldots & b_{1m}(t) \\ \ldots & \ldots & \ldots \\ b_{n1}(t) & \ldots & b_{nm}(t) \end{bmatrix}$$

$< x, y >$ will be used to denote the integral,

$$\int_{t_o}^{t_1} \sum_{i=1}^{n} x_i(t)\, y_i(t)\, dt \, .$$

$X(t)$ is the fundamental matrix solution of

$$\frac{dx_i}{dt} = \sum_{j=1}^{m} a_{ij}(t)\, x_j(t) \quad , \qquad i = 1, \ldots, n$$

which satisfies

$$X(t_o) = I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix} ,$$

$g(t)$ is the vector $B^*(t)\ X^{*-1}(t)\ X^*(t_1)\ X(t_1)\ C$

where the * denotes the transpose, C is the initial condition vector, and

$$Ru(t) = B^*(t)\ X^{*-1}(t)\ X^*(t_1)\ u(t) \int_{t_o}^{t_1} X(t_1)\ X^{-1}(s)\ B(s)\ u(s)\ ds .$$

Superscripts on vectors or scalars as $u^i(t)$, will denote the number of the iteration.

The procedure is as follows. Guess at $u(t) = u^1(t)$, $t_o \leq t \leq t_1$. Then choose

$$\bar{u}^2(t) = u^1(t) - \epsilon^1\ Z^1(t) ,$$

where

$$\epsilon^1 = \frac{< Z^1, Z^1 >}{< (R + kI)Z^1, Z^1 >} , \quad Z^1(t) = Ru^1(t) + g(t) .$$

If $\int_{t_o}^{t_1} ||\ \bar{u}^2(t)\ ||^2\ dt \leq M^2$

then let $u^2(t) = \bar{u}^2(t)$. Likewise, choose

$$\bar{u}^{i+1} = u^i - \epsilon^i\ Z^i$$

where

$$\epsilon^i = \frac{< Z^i, Z^i >}{< (R + kI)Z^i, Z^i >} , \quad Z^i = Ru^i + g ,$$

and choose

$$u^{i+1}(t) = \bar{u}^{i+1}(t) \text{ if } \int_{t_o}^{t_1} || \bar{u}^{i+1}(t) ||^2 \, dt \leq M^2 \ .$$

Stop the process at the j^{th} stage if

$$\int_{t_o}^{t_1} || u^{i+1}(t) - u^i(t) ||^2 \, dt$$

is less than some preassigned quantity.

Suppose that at the r^{th} stage we have

$$|| \bar{u}^r ||^2 = \int_{t_o}^{t_1} || \bar{u}^r(t) ||^2 \, dt > M^2 \ .$$

Then find a positive number k_r such that

$$\int_{t_o}^{t_1} || u^{r+1,k_r}(t) ||^2 \, dt = M^2 \ ,$$

where $\quad u^{r+1,k_r}(t) = u^r(t) - \epsilon^r Z^r \ ,$

$$\epsilon^r = \frac{< Z^r, Z^r >}{< (R+(k_r)I)Z^r, Z^r >} \ ,$$

$$Z^r = (R+(k_r)I)u^2 + g \ .$$

Choose $u^{r+1}(t) = u^{r+1,k_r}(t)$.

No conclusive results were obtained on the speed of the convergence of this technique. However, it is believed that it would be a worthwhile task to attempt to adapt this technique to solving the minimax problem.

REFERENCE

1. A. V. Balakrishnan, "An Operator-Theoretic Formulation of a Class of Control Problems and a Steepest Descent Method of Solution," J. S.I.A.M. Control, Series A, vol. 1, No. 2, 1963, pp. 109-127.

APPENDIX E

MINIMUM DRIFT AND MINIMUM LOAD CONTROL FOR MODEL VEHICLE NO. 2

E.1 Introduction

For large space vehicles where reduction in structure weight is of prime consideration it is difficult to provide aerodynamically stable air frames. In order to minimize weight and achieve effective stabilization in all phases of powered flight, within and without the atmosphere, modern day boosters are provided with swiveling engines. The control of the booster is accomplished by suitably adjusting the gimbal angle for satisfactory operation during the entire flight.

The design of control systems for present day vehicles is based on rigid body approximation. In practice this assumption is reasonably good, since it is possible to suppress the effect of bending and slosh in present day vehicles.

In designing a controller for the rigid vehicle there are two major points to be kept in mind.

(i) Control deflection angle has a hard constraint.

(ii) Aerodynamic pressures may break the vehicle.

Thus it is required to design a controller, such that under expected disturbances it is possible to steer the vehicle with the available thrust and control deflection angle, and bring it reasonably close to the desired trajectory at the end of the flight time, maintaining the aerodynamic loading within the structural limits.

There are two control schemes in use which come close to meeting the above requirements, namely, (i) Minimum Drift Control (MDC), (ii) Minimum Load Control (MLC). Here we shall study the behavior of MV2 under these schemes.

E.2 Rigid Body Dynamics of MV2

The nominal (reference) trajectory for MV2 is a gravity turn trajectory. It is the concern of guidance to give a command(normally pre-programmed) to follow the trajectory (in the absence of un-accounted-for disturbances). The control problem is to steer the vehicle close to this trajectory in the presence of disturbances by perturbing the command signal. Since we are interested in the control problem, it suffices to study the motion of the vehicle with reference to a gravity turn trajectory.

For small perturbations from the nominal, the rigid body equations can be written independently in three planes. Assuming a nominal gravity turn trajectory, the perturbation equations in the Yaw and pitch planes are identical. In either of these planes, the dynamics of the rigid vehicle is described by the following equations:

$$\ddot{\phi} + C_1\alpha + C_2\beta = 0 \tag{E.1}$$

$$\ddot{Z} = C_5\phi + C_3\alpha + C_4\beta \tag{E.2}$$

$$\alpha = \phi - C_7\dot{Z} + \alpha_W \tag{E.3}$$

The variable involved in Equations (E.1), (E.2), and (E.3) are defined in Appendix B.

E.3 Minimum Load Control

MV2 has its center of pressure ahead of its center of gravity and hence is inherently unstable. The vehicle can be artificially stabilized by feeding back a linear combination of attitude angle, its derivative and angle of attack. Let

$$\beta = a_0\phi + a_1\dot{\phi} + b_0\alpha \tag{E.4}$$

and let us analyze the feedback system.

Substituting (E.4) in (E.1) and (E.2) we get

$$\ddot{\phi} + a_1 C_2 \dot{\phi} + a_o C_2 \phi + (C_1 + b_o C_2)\alpha = 0 \tag{E.5}$$

$$\ddot{Z} = (C_5 + a_o C_4)\phi + a_1 C_4 \dot{\phi} + (C_3 + b_o C_4)\alpha \tag{E.6}$$

The characteristic equation of the feedback system, assumed time-invariant, is given by

$$\begin{aligned} CE = S^3 &+ [C_7(C_3 + b_o C_4) + a_1 C_2]S^2 \\ &+ [a_1 C_7(C_2 C_3 - C_1 C_4) + (a_o + b_o)C_2 + C_1]S \\ &+ C_7[a_o(C_3 C_2 - C_1 C_4) - C_5(C_1 + b_o C_2)] = 0 \end{aligned} \tag{E.7}$$

By properly choosing the values of a_o, a_1 and b_o it may be possible to locate the roots of the CE such that in quasi-steady state the derivatives of attitude angle and drift-rate become negligible. Assuming that this can be done, Equations (E.4), (E.5) and (E.6) reduce to (E.8), (E.9) and (E.10) respectively.

$$\beta = a_o \phi + b_o \alpha \tag{E.8}$$

$$a_o C_2 \phi + (C_1 + b_o C_2)\alpha = 0 \tag{E.9}$$

$$(C_5 + a_o C_4)\phi + (C_3 + b_o C_4)\alpha = 0 \tag{E.10}$$

In (E.8), (E.9) and (E.10):

$$\beta = \beta_{\text{quasi ss}}$$

$$\phi = \phi_{\text{quasi ss}}$$

$$\alpha = \alpha_{\text{quasi ss}}$$

Solving for $\phi_{\text{quasi ss}}$, $\beta_{\text{quasi ss}}$ and $\alpha_{\text{quasi ss}}$ from Equations (E.3), E.8), (E.9) and (E.10) we get:

$$\phi_{\text{quasi ss}} = \frac{(C_1 + C_2 b_o)\,(C_7\dot{Z} - \alpha_W)}{C_1 + C_2\,(a_o + b_o)} \tag{E.11}$$

$$\alpha_{\text{quasi ss}} = \frac{a_o C_2\,(\alpha_W - C_7\dot{Z})}{C_1 + C_2\,(a_o + b_o)} \tag{E.12}$$

$$\beta_{\text{quasi ss}} = \frac{a_o C_1 (\alpha_W - C_7\dot{Z})}{C_1 + C_2 (a_o + b_o)} \tag{E.13}$$

Now if $a_o = 0$, then

$$\alpha_{\text{quasi ss}} = 0, \qquad \beta_{\text{quasi ss}} = 0$$

But the bending moment for Model Vehicle 2 at a point 90 meters from the bottom end is given by

$$\text{B.M.} = (1.845\alpha + 12.38\beta) \times 10^6 \tag{E.14}$$

Thus if the attitude feedback is zero, then in the quasi-steady state, the bending moment or the aerodynamic loading goes to zero. Hence

$$\beta = a_1\dot{\phi} + b_o\alpha \tag{E.15}$$

is called minimum load control (MLC).

It should be emphasized here that the above conclusions are based on rigid body assumption and are essentially true in the steady state when all the closed-loop poles are in the left half plane. It has been observed [1] that with MLC, even in quasi-steady state, the peak value of the control deflection angle β and the angle of attack α are smaller as compared to their values in other modes of control. This implies that the bending moment, which is a function of β and α, would also be small. Later in this appendix some computational results for MV2 (rigid body approximation) with MLC and MDC are given. Here it is found that the peak value of α with MLC is half of the peak value with MDC. However the peak values of β and the bending moment with MLC are 97.4% and 80.5% respectively of the peak values with MDC.

Next we consider the influence of a_1 and b_o on the system behavior.

E.4 <u>Effect of a_1 and b_o on System Response</u>

(1) <u>Let $b_o = 0$, vary a_1</u>

For MLC with $b_o = 0$, the characteristic equation of the system reduces to

$$S^3 + (C_7C_3 + a_1C_2)S^2 + (a_1C_2C_3C_7 - a_1C_1C_4C_7 + C_1)S - C_1C_5C_7 = 0 \tag{E.16}$$

This can be rewritten as:

$$1 + \frac{a_1}{C_2} \cdot \frac{S + \frac{C_2}{C_7}(C_2C_3 - C_1C_4)\,S}{(S^3 + C_3C_7S^2 + C_1S - C_1C_5C_7)} = 0 \tag{E.17}$$

For MV2 (at t = 80 sec.) this reduces to

$$1 + \frac{a_1}{.448} \cdot \frac{S\,(S + 0.01443)}{(S^3 + .01073S^2 - .0791\,S + .00318)} = 0 \tag{E.18}$$

or

$$1 + \frac{a_1}{.448} \cdot \frac{S(S + 0.01443)}{(S - 0.2525)\,(S - 0.04145)\,(S + 0.3047)} = 0 \tag{E.19}$$

Now a root locus can be plotted with a_1 as parameter. The roots of the denominator in (E.19) are the open loop poles of the system. The root locus is given in Figure E.1.

From the root locus in Figure E.1 it is clear that with attitude rate feedback alone it is possible to stabilize the system. In case of MV2 for $a_1 = 15$ or larger, all the closed loop poles lie in the left half plane. However it is not feasible to do this in practice because of the zero at the origin. With the zero at the origin, the the low frequency response to rate commands is very poor, which means the system

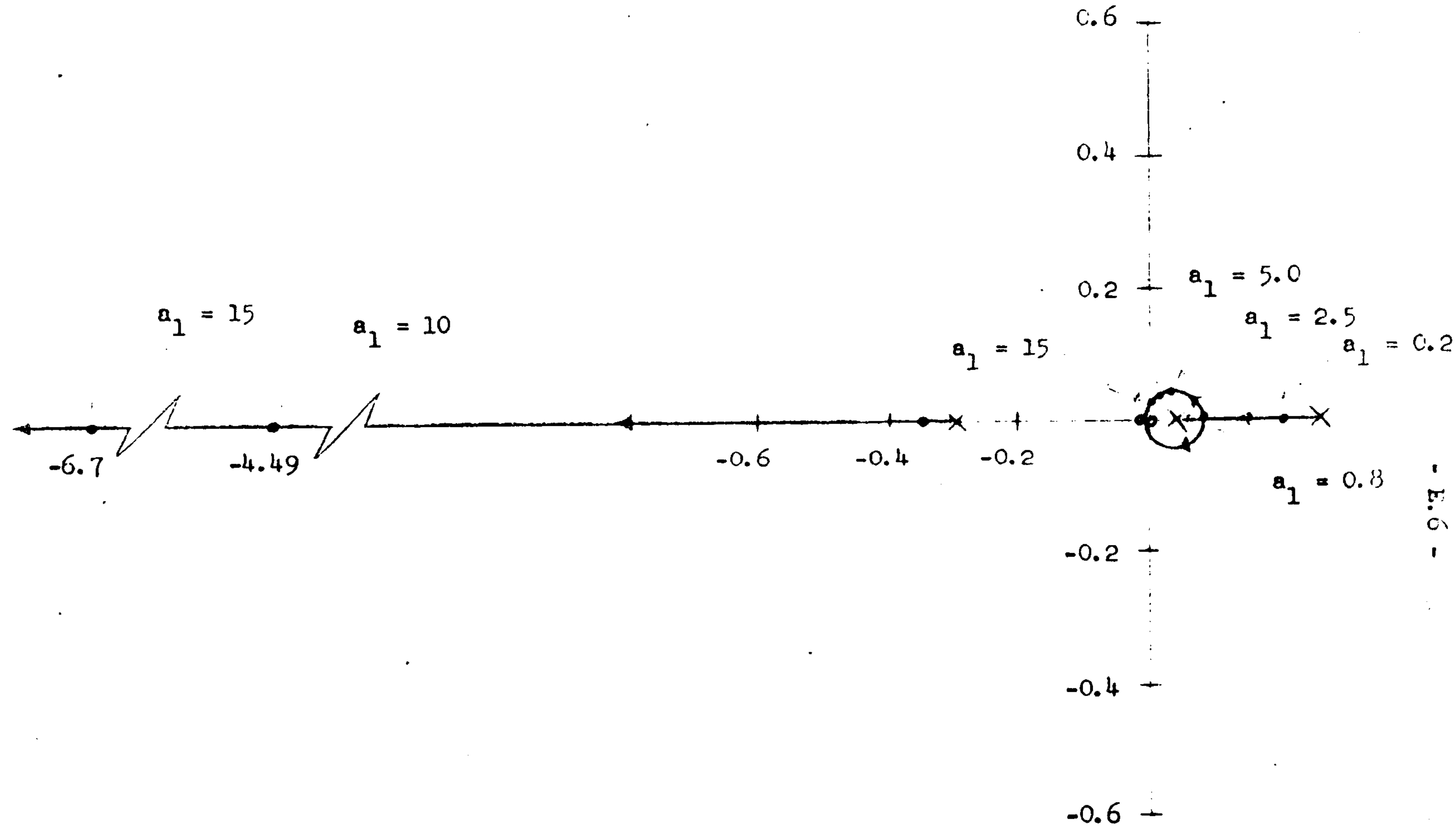


Figure E.1

Root Locus for $a_o = 0$, $b_o = 0$, a_1 Increasing would not respond well to guidance maneuvers.

(2) Vary b_o, fix a_1-

To consider the case with a_1 fixed and b_o varying, let us rewrite the characteristic equation (E.7) as follows:

$$1 + b_o \frac{C_4C_7S^2 + C_2S - C_2C_5C_7}{\{S^3 + (C_3C_7 + a_1C_2)S^2 + [a_1C_7(C_2C_3 - C_1C_4) + C_1]S - C_1C_5C_7\}} = 0 \tag{E.20}$$

With a_1 fixed at 2.5, Equation (E.20) reduces to

$$1 + \frac{b_o}{C_4C_7} \cdot \frac{(S + 21.45)\ (S - 0.04)}{(S + 1.186)\ (S - 0.0276 \pm j\ 0.04386)} \tag{E.21}$$

The root locus corresponding to Equation (E.21) for increasing b_o is given in Figure E.2.

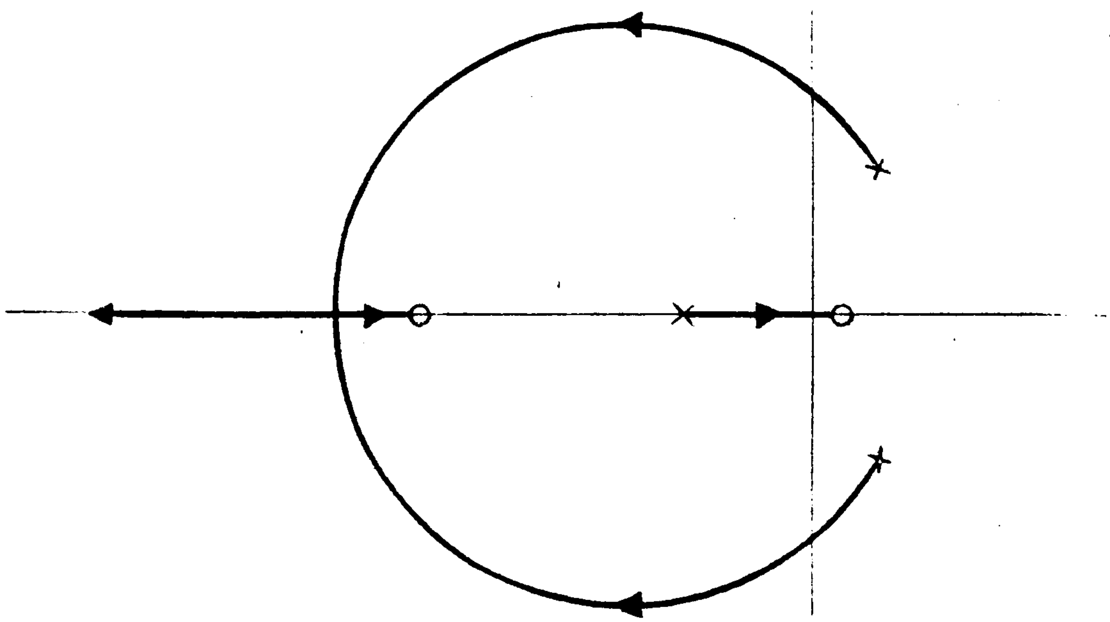


Figure E.2

Root Locus for $a_o = 0$, $a_1 = 2.5$, b_o Increasing

Proceeding along the same lines as above, a root locus can be plotted for simultaneous variation of a_1 and b_o, when they are related by a constant. The root locus corresponding to the case when $a_1 = K\ b_o$, and b_o increasing is sketched in Figure (E.3)

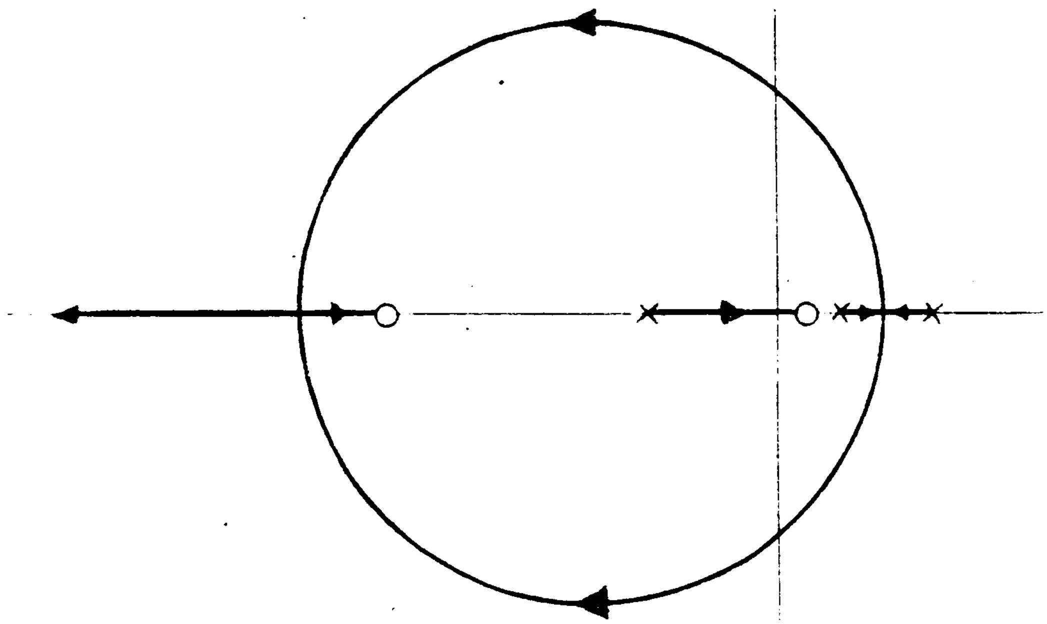


Figure E.3

Root Locus for $a_o = 0$, $a_1 = 0$, b_o Increasing

It can be seen from Figures (E.2) and (E.3) that for small values of b_o, the root representing the lateral motion of the center of gravity of the vehicle is stable and may become unstable if b_o is large enough. From the characteristic equation (E.7) it can be seen that if the constant term is positive, the root corresponding to the lateral motion of the vehicle is stable and if it is negative then the root is unstable. Thus if $b_o = - \frac{C_1}{C_2}$, the path root is at the origin, and this corresponds to the case of MDC, since under this condition the lateral drift is minimum. If $b_o < -\frac{C_1}{C_2}$ the path root is stable and the vehicle drifts with the wind. Whereas for $b_o > -\frac{C_1}{C_2}$, the vehicle turns its nose into the wind to such a degree that a positive accelaration against the wind takes place and the path root is unstable. From the above discussion it appears that by a proper choice of b_o, MLC and MDC can be employed simultaneously. But, as will be shown shortly, this is not feasible.

We digress here to determine the rigid body natural frequency and the damping of rotatory motion.

Substituting for α and β in Equation (E.1), from Equation (E.3) and (E.15), it follows that

$$\ddot{\phi} + C_1 (\phi - C_7\dot{Z} + \alpha_W) + C_2[a_1\dot{\phi} + b_o (\phi - C_7\dot{Z} + \alpha_W)] = 0$$

or

$$\ddot{\phi} + a_1C_2\dot{\phi} + (C_1 + b_oC_2)\phi = (C_1C_7\dot{Z} + b_oC_2C_7\dot{Z} - C_1\alpha_W - b_oC_2\alpha_W)$$

Neglect $\dot{Z}$ and consider the homogeneous equation

$$\ddot{\phi} + a_1C_2\dot{\phi} + (C_1 + b_oC_2)\phi = 0 \tag{E.22}$$

From Equation (E.22) by inspection

$$\omega_n = \sqrt{C_1 + b_oC_2} \tag{E.23}$$

$$\xi = \frac{a_1C_2}{2\sqrt{C_1 + b_oC_2}} \tag{E.24}$$

Equations (E.23) and (E.24) give approximate expressions for the natural frequency and the damping of rotatory motion of the rigid vehicle. The damped natural frequency is given by

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = \sqrt{(C_1 + b_o C_2)} \sqrt{1-\xi^2} \tag{E.25}$$

Now returning to the discussion of simultaneous MLC and MDC we note that the rigid body ω_n goes to zero. This is **undesirable because** small bandwidth leads to sluggish response.

E.5 Computational Results

For $\xi = 0.7$ and $f_n = 0.2$ cps, MV2 response under MLC in the presence of disturbance α_w supplied by MSFC (see Figure 5.13a) is evaluated and presented in Figure E.4. For computational check and comparative study, similar results under MDC are also generated (Figure E.5).

(1) Minimum Drift Control

$$f_n = \frac{\omega_n}{2\pi} = 0.2 \text{ cps}$$

$$\xi = 0.7$$

(Results for the above values of f_n and ξ have been provided by MSFC.)

The corresponding values of control coefficients are: $a_o = 2.489$; $b_o = 0.8876$, $a_1 = 3.945$. The drift at the end of 23 sec. is 3.7 m/sec. The peak value of the B.M. is 0.613×10^6 kgm (maximum allowable value is 2.24×10^6 kgm) and the maximum value of attitude angle is 1.8^o. The results, except for the drift rate, compare fairly well with those provided by MSFC. According to MSFC results the drift rate $\dot{Z}$ varies between $\pm$ 0.5 m/sec., with a final value of -0.5 m/sec. Whereas according to the results presented here, the drift rate does not change sign and has a final value of 3.7 m/sec. The difference may be due to any possible variations in the values of the coeffieients C_1 through C_7 (see Appendix B).

(2) Minimum Load Control

As stated above, the system response to this control mode is found for $\xi = 0.7$ and $f_n = 0.2$ cps. The corresponding values of a_1 and b_o are:

$$a_1 = 3.94, \qquad b_o = 3.68$$

Unlike the case of MDC, the drift in the case of MLC is 47 meters/sec. However the peak value of the bending moment is 0.4931×10^6 kgm as compared to 0.613×10^6 in the other case. Because of a pole in the right half plane, the attitude angle is much larger —12.5°. The attitude rate in the quasi-steady state (no shear or gusts in α_W) is fairly small, as assumed in the derivation of MLC.

In general MLC does result in small bending moments, provided the disturbance is constant or varying slowly with time and altitude. In case of high shears there is a tendency for the bending moment under MLC to take higher values as compared to other modes. From Figures E.4 and E.5 we see that the bending moment at 68.5 seconds is 0.33×10^6 kgm and 0.21×10^6 kgm under MLC and MDC respectively. That is the bending moment is larger under MLC. Thus if there are high frequency changes in the wind in the high aerodynamic pressure region, then MLC mode is not appropriate.

It appears that guidance induced maneuvers are essential if MLC is used. These maneuvers would correct for excessive departures from the nominal trajectory incurred while using MLC.

Thus far we have studied MLC for a rigid body. We next consider the effect of "tail-wags-dog," "slosh" and "bending." It should be emphasized that the advantages and disadvantages of MLC as discussed so far are strictly with respect to a rigid vehicle with no tail-wags-dog or slosh.

E.6 Effect of Tail-Wags-Dog

In rigid body approximation, we assumed the control force normal to the vehicle axis to be $\beta \times$ (control thrust). But if the inertial reaction force (due to $\ddot{\beta}$) is considered, it can be shown that the normal control force experiences an antiresonance. This antiresonance is called the "tail-wags-dog" phenomenon. Taking inertial reaction force into account, Equation (E.1) changes to

$$\ddot{\phi} + C_1\alpha + C_2\beta + C_5 \frac{S_E}{I}\beta + \frac{I_E + S_E(\alpha_{CG} - x_\beta)}{I} \qquad \text{(E.26)}$$

For MV2

$$C_5 \frac{S_E}{I} = 0.000097$$

and

$$\frac{I_E + S_E\,(x_{CG} - x_\beta)}{I} = 0.000192$$

Substituting the numerical values in the coefficients of (E.26)

$$\ddot{\phi} - 0.0791\alpha + (.448 + .000097)\beta + (0.000192)\ddot{\beta} = 0$$

or

$$\ddot{\phi} - 0.0791\,\alpha + 0.000192\,(\ddot{\beta} + 2340\beta) = 0$$

Thus the frequency of antiresonance is

$$\omega = \sqrt{2340} = 48.4 \text{ rad./sec.}$$

or $\qquad f = 7.6$ cps.

Since the antiresonance frequency is fairly high compared to the desirable control frequency, this phenomenon is of negligible consequence under MLC.

In terms of root locus plot, tail-wags-dog phenomenon introduces a pair of zeros on the imaginary axis, at $\pm j\,48.4$.

E.7 Effect of Propellant Sloshing

In large boosters using liquid propellants, sloshing or splashing of propellants against the walls of the booster is a problem of serious concern. The sloshing phenomenon is analyzed by a mass-spring analogy. The effect of sloshing is to introduce a dipole of the form $\frac{S^2 + \omega_i^2}{S^2 + \omega_i'^2}$

where ω_i is the frequency of oscillation of the sloshing mode at i^{th} station. If damping is taken into account, the dipoles become complex in nature. The sloshing poles are non-dominant because of their close proximity to sloshing zeros.

E.8 Effect of Bending

Similar to the effect of slosh, bending introduces dipoles in the plant transfer function. For MV2, the first bending frequency is close to the slosh frequency.

Bending creates the most critical problems associated with closed-loop stability. The attitude gyros sense the vehicle bending modes in addition to rigid body attitude changes. The change in β is due to the bending modes and to true change in the attitude angle of the rigid body; this in turn may excite the bending modes further and an eventual instability may result.

E.9 Discussion and Conclusion

The results obtained for rigid body approximation to MV2 under MLC are impressive. The peak value of the bending moment is well within the given constraints. Tail-wags-dog may be entirely neglected from the problem of analysis and design for the rigid body. However low slosh and first bending frequencies are a matter of concern.

A rule of thumb is to have the control frequency about 1/4 of the first bending mode frequency. However in the case of MV2 with a control frequency of 0.2 cps, the ratio between the control frequency and the first bending frequency (0.36 cps) is 1:1.8. Hence for successful operation of the flexible vehicle a good filter needs to be designed, so that only the true attitude rate and angle of attack may be fed back.

From the results in Figures E.4 and E.5 the following conclusion can be made: With either mode of control, MLC or MDC, the peak value of the bending moment is well below the design limit 2.24 kgm. A reduction of 20% in the peak value of the bending moment under MLC does not seem to be of much value when one considers the large drift and attitude deviation from the nominal trajectory that results under MLC. If MLC is used, guidance induced maneuvers are essential to account for excessive departure from the nominal trajectory.

REFERENCES

1. Geissler, E. D., "Problems in Attitude Stabilization of Large Guided Missiles," Aerospace Engineering, October 1960.

2. Hoelker, R. F., "Theory of Artificial Stabilization of Missiles and Space Vehicles with Exposition of Four Control Principles," Technical Note D-555, NASA, Washington, June 1961.

3. Johnson, G. W., F. G. Kilmer, "MARP Report on Ballistic Missile Flight Control Theory," IBM 60-504-48, IBM FSD Space Guidance Center, Owego, New York, January 26, 1961.

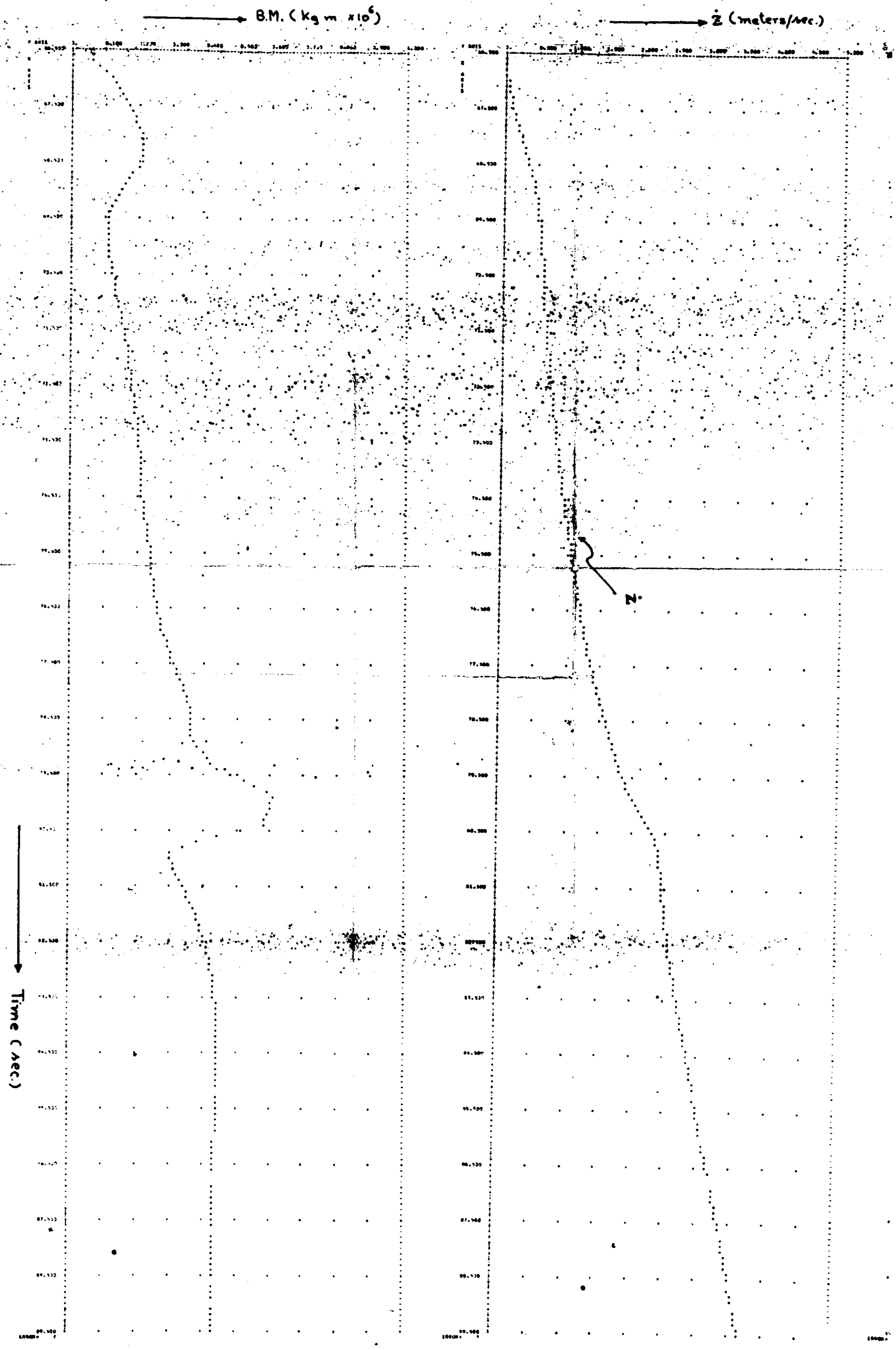
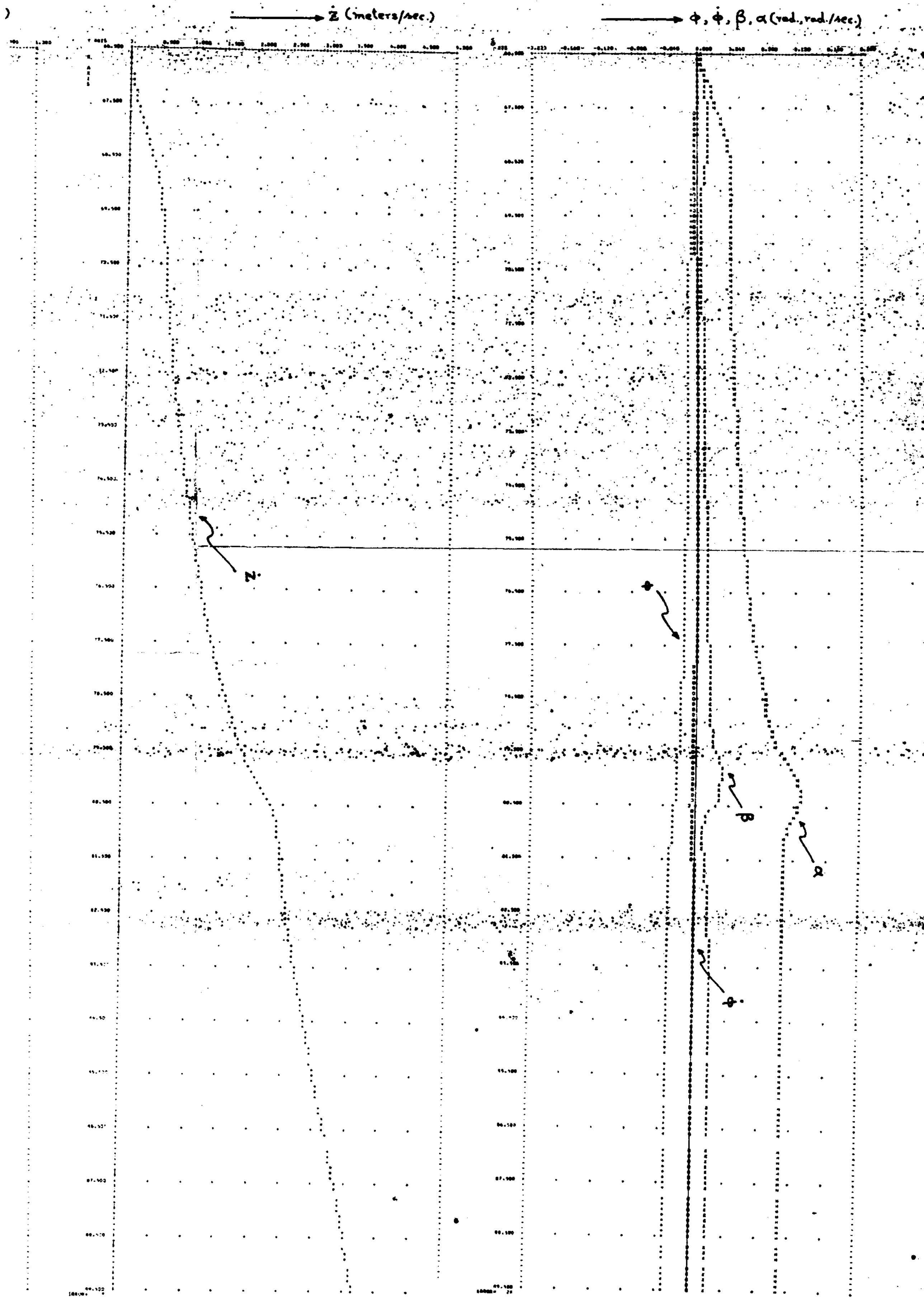


Fig E.5 Response of MV 2 under Minimum Drift Control.



→ ż (meters/sec.)
→ φ, φ̇, β, α (rad., rad./sec.)
ż
φ
β
α
φ̇

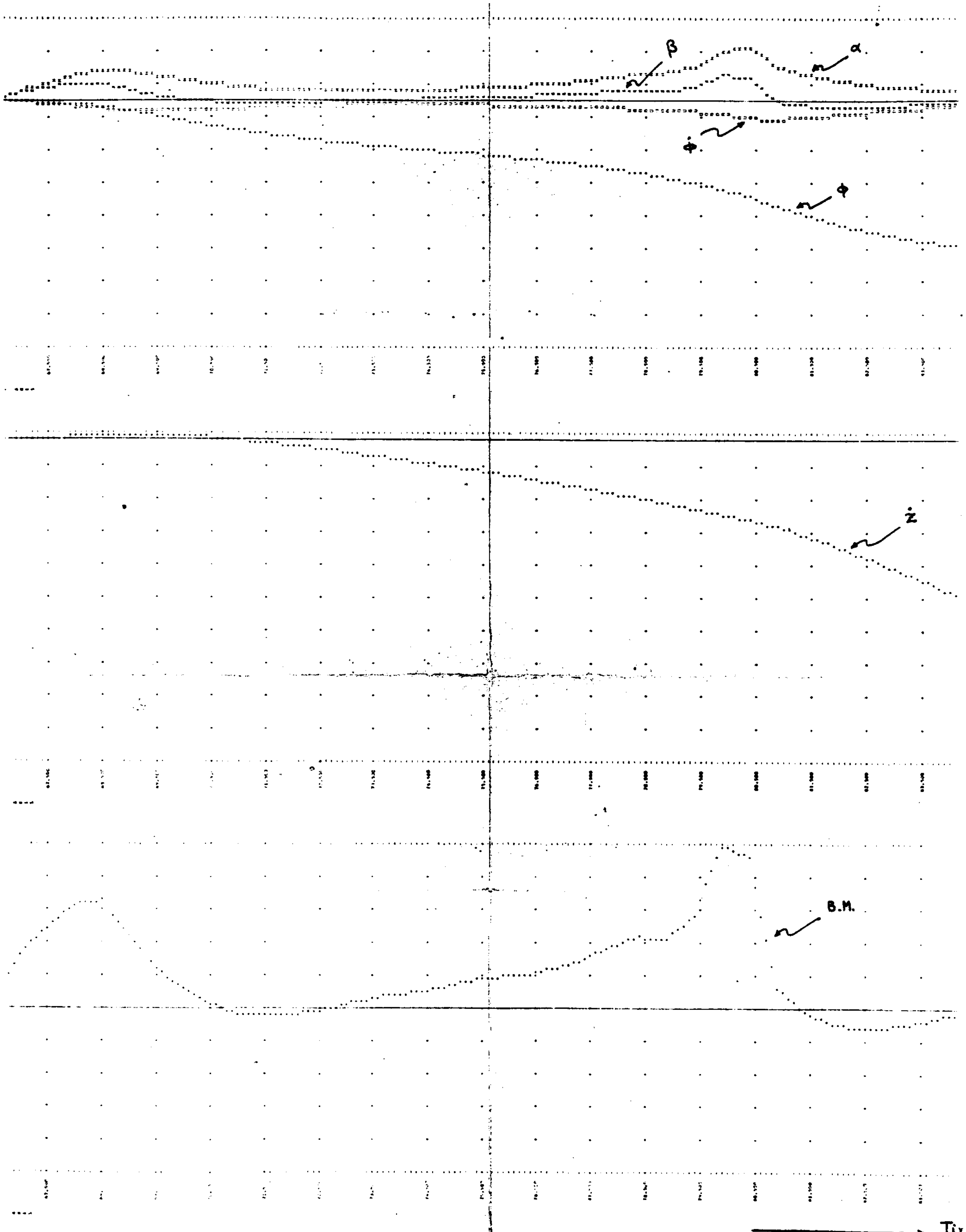
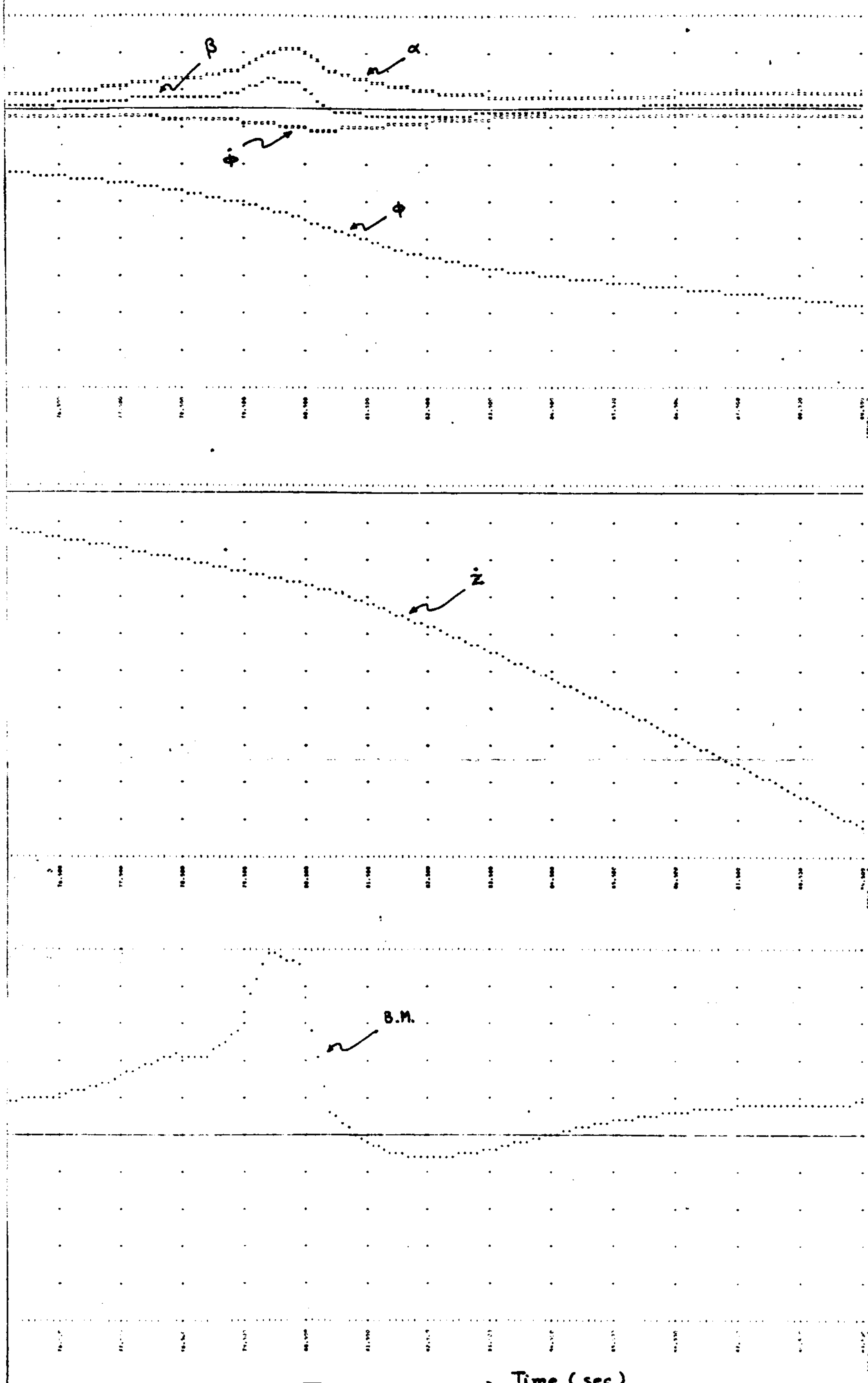


Fig. E.4 Response of MV2 under Minimum Load Control



f MV 2 under Minimum Load Control.